Physical Principles in Biology
Biology 3550
Fall 2016

Lecture 14

A Deviation: From a Random Walk to Error Analysis

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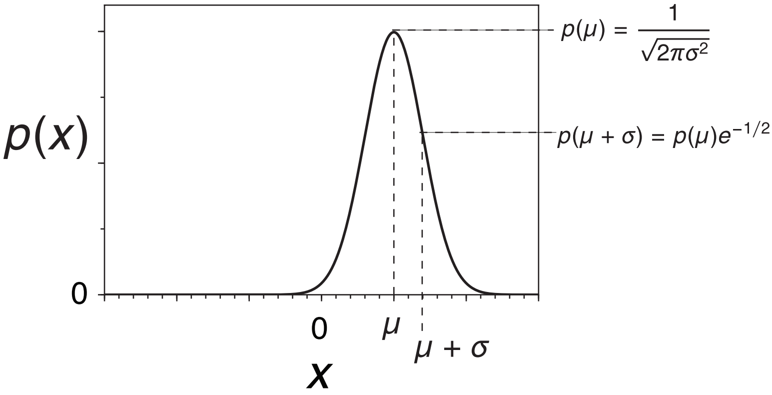
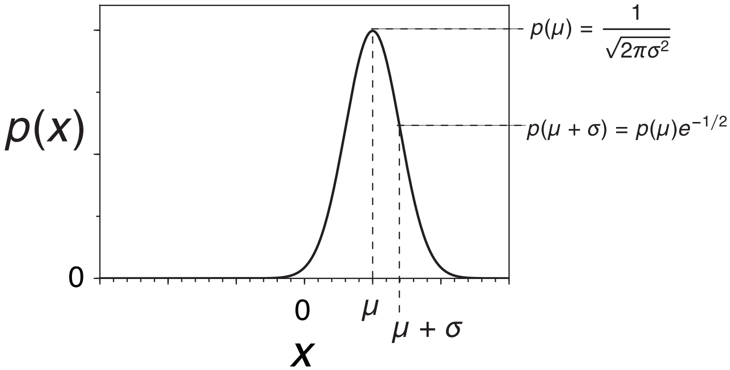
The General Form of the Gaussian PDF

- $p(x) = \sqrt{\frac{1}{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}$



- μ is the expected value of x, $\langle x \rangle$, and represents the displacement of the distribution from 0.

The General Form of the Gaussian PDF

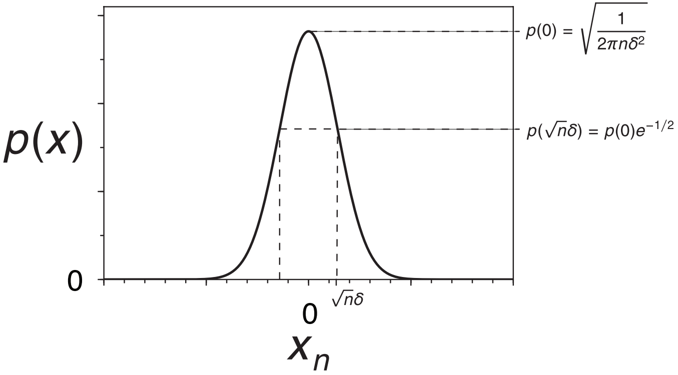


- σ^2 is called the "variance" and is the mean-square deviation from the mean of values drawn from the distribution: $\sigma^2 = \langle (x-\mu)^2 \rangle$
- σ is called the "standard deviation" and is the root-mean-square deviation from the mean: $\sigma = \sqrt{\langle (x-\mu)^2 \rangle} = \mathrm{RMS}(x-\mu)$

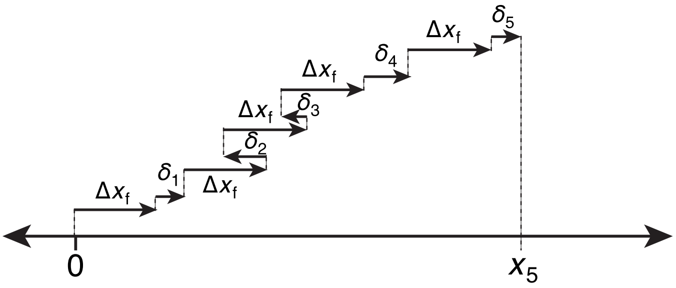
Gaussian Distribution for 1-dimensional Random Walk with Distribution of Step Lengths, δ_i

- Assume: n steps, $\langle \delta_i \rangle = 0$ and $\langle \delta_i^2 \rangle = \delta^2$.
- From before: $\langle x_n \rangle = 0$ and $\langle x_n^2 \rangle = n\delta^2$.
- Gaussian distribution:

$$p(x_n) = \sqrt{\frac{1}{2\pi n\delta^2}}\, e^{-x^2/(2n\delta^2)}$$



A New Kind of Random Walk

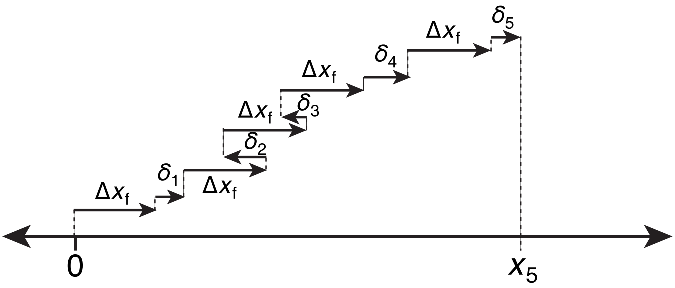


- Each step, i, has two parts:
  1. A fixed displacement of Δx_f, always of the same magnitude and sign.
  2. A random displacement of δ_i, such that:

$$\langle \delta_i \rangle = 0$$
$$\langle \delta_i^2 \rangle = \delta^2$$

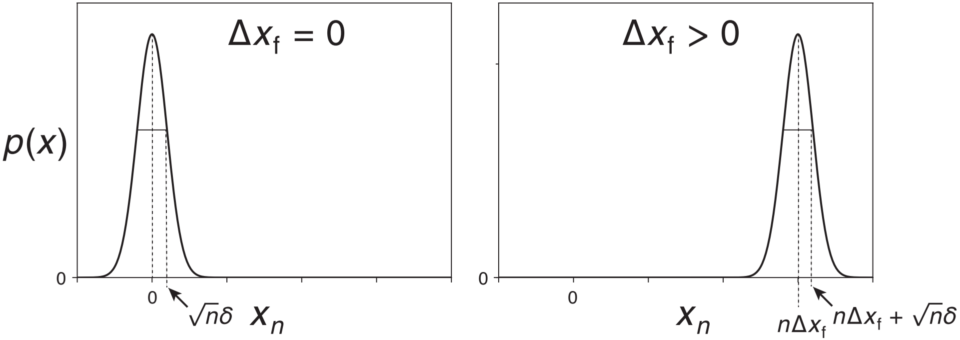
- Each random walk consists of n two-step steps.

A New Kind of Random Walk



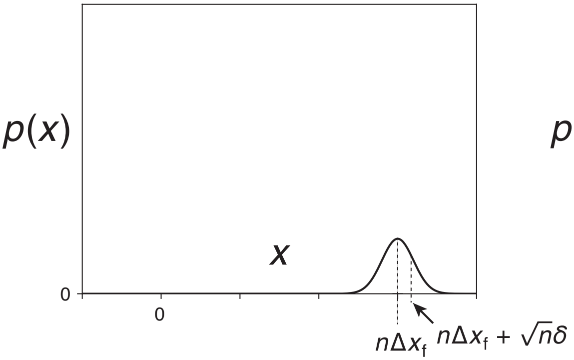
- If $\Delta x_f = 0$, and we do a large number, N, of walks:
  - It's just like our old random walks.
  - $\langle x_n \rangle = 0$
  - $\langle x_n^2 \rangle = n\delta^2$
  - The values of x_n have a normal distribution, with $\mu = 0$ and $\sigma = \sqrt{n}\delta$.

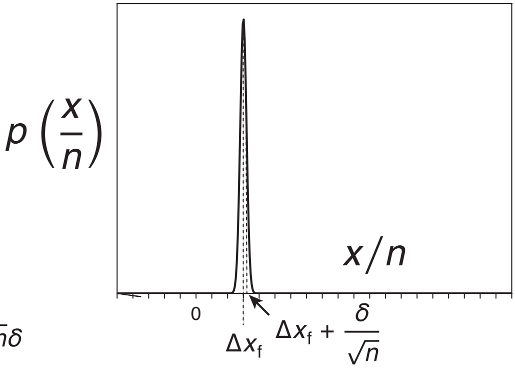
Distribution of Endpoints, x_n, for N Walks



- $\langle x_n \rangle = n\Delta x_f$
- $\langle (x_n - n\Delta x_f)^2 \rangle = n\delta^2$
- A normal distribution, with $\mu = n\Delta x_f$ and $\sigma = \sqrt{n}\delta$.
- As n increases, the distribution moves to the left *and* spreads out.

Suppose that we divide x by n





- $\langle x_n/n \rangle = \Delta x_\mathrm{f}$
- $\langle (x_n/n - \Delta x_\mathrm{f})^2 \rangle = \delta^2/n$
- A normal distribution, with $\mu = n\Delta x_\mathrm{f}$ and $\sigma = \delta/\sqrt{n}$.
- The longer the walk, the narrower the distribution of x/n.

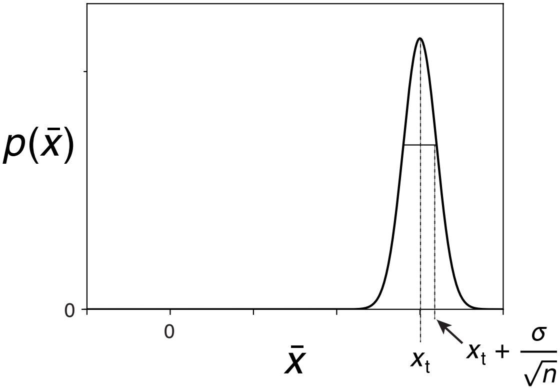
Something Similar to Our Funny Random Walk

- Make n repeated measurements of some physical quantity, x, such as the mass of an object.
- Each measurement, x_i, is the sum of:
  - The "true" value of the measurement, x_{t}.
  - A random error, δ_i, with $\langle\delta\rangle = 0$ and a standard deviation, $\sigma = \mathrm{RMS}(\delta)$.
- Sum all of the measured values:

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} (x_{\mathrm{t}} + \delta_i)$$

- Divide the sum by the number of measurements, n.
  Call this $\bar{x}$.
- How would we interpret the result: $\bar{x} = \left(\sum x_i\right)/n$?
- What would we expect to find if we were to repeat the series of n measurements a large number of times?

Distribution of Averages of Experimental Measurements



- The most probable value of $\bar{x}$ is x_t, the “true” value of x.
- The width of the distribution is proportional to σ, the standard deviation of the experimental errors.
- The width of the distribution decreases with $\sqrt{n}$.