

Physical Principles in Biology
Biology 3550
Fall 2016

Lecture 29

Chemical Thermodynamics and Metabolism

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The Free Energy Change for a Chemical Reaction



- $\Delta G = \Delta H - T\Delta S_{\text{sys}}$
- ΔG° : ΔG when all concentrations are 1 M (or 1 atm for gasses)
- Two equivalent interpretations:
 - Free energy change of converting one mole of reactants to one mole of products, under standard state conditions and in a volume so large that concentrations don't change significantly.
 - Free energy change for the change of concentrations from standard state concentrations to equilibrium concentrations.

The Free Energy Change for a Chemical Reaction



- ΔG for other concentrations:

$$\Delta G = \Delta G^\circ + RT \ln \frac{[B]}{[A]}$$

- Reaction at equilibrium:

$$\Delta G = 0 = \Delta G^\circ + RT \ln \frac{[B]_{\text{eq}}}{[A]_{\text{eq}}}$$

$$\Delta G^\circ = -RT \ln K_{\text{eq}}$$

The Relationship Between Free Energy and the Equilibrium Constant

$$A \rightleftharpoons B \quad K_{\text{eq}} = \frac{[B]_{\text{eq}}}{[A]_{\text{eq}}} \quad \Delta G^\circ = -RT \ln K_{\text{eq}}$$

- If $K_{\text{eq}} > 1$:
 - Reaction favors B over A.
 - $\Delta G^\circ < 0$

- If $K_{\text{eq}} < 1$:
 - Reaction favors A over B.
 - $\Delta G^\circ > 0$

- If $K_{\text{eq}} = 1$:
 - A and B have equal free energies.
 - $\Delta G^\circ = 0$

Reactions with More than One Reactant or Product

- A reaction with two reactants and two products: $aA + bB \rightleftharpoons cC + dD$
a, b, c and d represent stoichiometry

- The equilibrium constant:

$$K_{\text{eq}} = \frac{[C]_{\text{eq}}^c [D]_{\text{eq}}^d}{[A]_{\text{eq}}^a [B]_{\text{eq}}^b}$$

Products over reactants! (Mrs. Freeman)

- The reaction quotient:

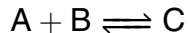
$$Q = \frac{[C]^c [D]^d}{[A]^a [B]^b}$$

When concentrations are not necessarily at equilibrium.

- $\Delta G = \Delta G^\circ + RT \ln Q$

Clicker Question #1

For the reaction:



What are the units of the equilibrium constant?

1 M

2 No units

3 M^{-1}

From Two Molecules to One

- A reaction with two reactants and one products: $A + B \rightleftharpoons C$
- The equilibrium constant:

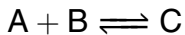
$$K_{\text{eq}} = \frac{[C]_{\text{eq}}}{[A]_{\text{eq}}[B]_{\text{eq}}}$$

- Some problems:
 - The equilibrium constant has units of inverse concentration.
 - The numerical value of K_{eq} will depend on the units that we use for concentration
 - What happens when we try to calculate ΔG° ?

$$\Delta G^\circ = RT \ln K_{\text{eq}}$$

Logarithms are functions of pure numbers. How do we take a logarithm of a value with units?

From Two Molecules to One



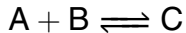
■ Defining standard states is important!

- Standard state concentration is usually defined as 1 M
- For thermodynamic purposes, refine the definition of the equilibrium constant:

$$K_{\text{eq}} = \frac{([C]_{\text{eq}}/1 \text{ M})}{([A]_{\text{eq}}/1 \text{ M})([B]_{\text{eq}}/1 \text{ M})}$$

- Now, K_{eq} is dimensionless, and we don't have to feel guilty about taking the logarithm.
- But, value of ΔG depends on choice of standard state concentrations. Why?

The Entropy Change for the Association of Two Molecules



- The entropy change for one mole each of A and B converted to C:

$$\Delta S = -R \ln (V(L) \times 10^{23})$$

(calculated from the Boltzmann entropy equation in Lecture 26)

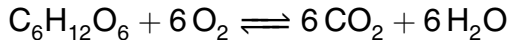
- For a given number of molecules, ΔS depends on the volume!
- The bigger the volume, the bigger the entropy loss.
- ΔG° , defined with specific concentration units, contains within it the entropy loss at the standard state concentration.

Chemical Energy



- What does “chemical energy” mean?

An Energetic Chemical Reaction, Under Some Conditions



- Oxidation of glucose (or other hexose) by molecular oxygen.
- Free energy change depends on concentrations.
- Standard state conditions: 1 M glucose, 1 atm O_2 and CO_2

- $\Delta G^\circ = -2.7 \times 10^6 \text{ J/mol} = -2,700 \text{ kJ/mol}$
- Equilibrium constant:

$$K_{\text{eq}} = \frac{[\text{CO}_2]_{\text{eq}}^6}{[\text{C}_6\text{H}_{12}\text{O}_6]_{\text{eq}}[\text{O}_2]_{\text{eq}}^6} = e^{-\Delta G^\circ/(RT)} \approx 10^{473} \text{ M}^{-1}$$

(Water is ignored in free energy change calculation)

- Extremely favorable!
- But, this assumes 1 atm O_2 and CO_2 .
- Reaction only became favorable about 2.5 billion year ago, when oxygen became abundant on earth.

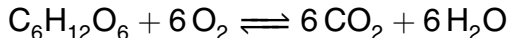
Nutritional Calories

- Measured as the heat for complete combustions with excess O_2 at constant volume (no work).

“bomb calorimeter”

- Since, $w = 0$, $q = \Delta E$
- For glucose, value is $4 \text{ kcal/g} = 175 \text{ kJ/mol}$, vs $\Delta G^\circ = 2,700 \text{ kJ/mol}$.
- Why are these numbers so different?

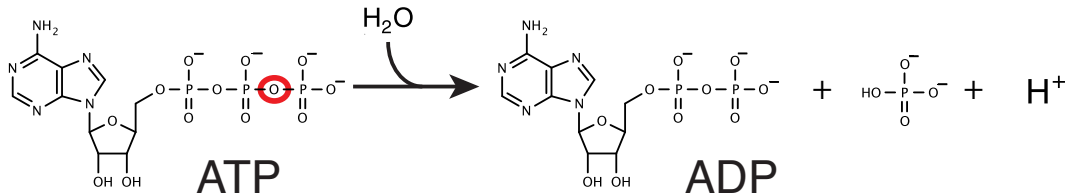
$$\Delta G = \Delta H - T\Delta S$$



There is a large increase in entropy, 7 molecules are converted to 12.

- Nutritionists estimate the amount of work required to offset the metabolism of a given number of nutritional calories.

Another “High-energy” Reaction



- $\Delta G^\circ = -30 \text{ kJ/mol}$
- ATP serves as an “energy currency” in metabolism.
- Is there a “high-energy bond”?

Clicker Question #2

What is the equilibrium constant For the reaction?



1 $\approx 10^{-5} \text{ M}$

2 $\approx 10^{-3} \text{ M}$

3 $\approx 10 \text{ M}$

4 $\approx 10^3 \text{ M}$

5 $\approx 10^5 \text{ M}$

$$R = 8.314 \text{ J}/(\text{K} \cdot \text{mol})$$

$$T = 298 \text{ K}$$

Calculation of Equilibrium Constant from ΔG°

$$\Delta G^\circ = -30 \text{ kJ/mol} \quad R = 8.314 \text{ J/(K} \cdot \text{mol)} \quad T = 298 \text{ K}$$

$$\Delta G^\circ = -RT \ln K_{\text{eq}}$$

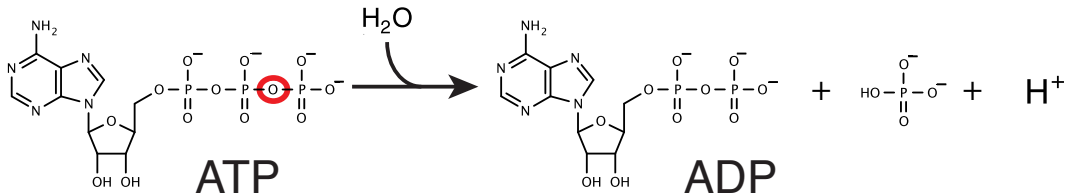
$$\ln K_{\text{eq}} = -\Delta G^\circ / (RT)$$

$$K_{\text{eq}} = e^{-\Delta G^\circ / (RT)}$$

$$\begin{aligned} -\Delta G^\circ / (RT) &= 30 \text{ kJ/mol} / (8.314 \text{ J/(K} \cdot \text{mol)} \cdot 298 \text{ K}) \\ &= 30 \times 10^3 \text{ J/mol} / 2.48 \times 10^3 \text{ J/mol} = 12 \end{aligned}$$

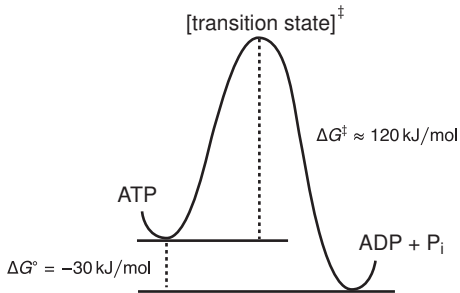
$$K_{\text{eq}} = e^{12} = 1.8 \times 10^5$$

ATP Hydrolysis



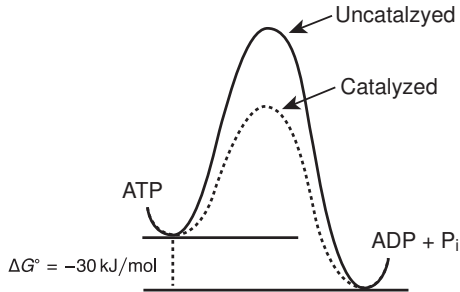
- $\Delta G^\circ = -30 \text{ kJ/mol}$
- Why is the reaction so favorable?
 - High density of negative phosphate charges is reduced.
 - More resonance stabilization in ADP and P_i .
 - More favorable interaction with water by ADP and P_i .

Kinetics of ATP Hydrolysis



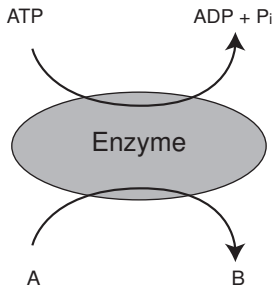
- Half-time is ≈ 20 days at neutral pH and 60°C .
- Transition state is a high energy state with equal probability of breaking down in either direction.
- Reaction rate is proportional to probability of forming the transition state.
- See the lobby of the Henry Eyring Building!

Catalysis of ATP Hydrolysis



- Enzymes catalyze chemical reactions by lowering transition-state energy.
- Enzymes create micro-environments that favor forming the transition state.
- Simply catalyzing ATP hydrolysis is not useful!

Enzymatic Coupling



- Enzyme mechanistically couples reactions.
- $\text{ATP} \rightarrow \text{ADP} + \text{P}_i$ can't occur without $\text{A} \rightarrow \text{B}$
- $\text{A} \rightarrow \text{B}$ can't occur without $\text{ATP} \rightarrow \text{ADP} + \text{P}_i$
- Coupled "reaction" can be physical motion or transport across membranes.