Physical Principles in Biology
Biology 3550
Fall 2016

Lecture 3

More on Dimensions and Units

Volumes and concentrations

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©David P. Goldenberg
University of Utah
goldenberg@biology.utah.edu

Clicker Question #1

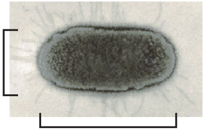
How many hydrogen ions (H^+) are in a typical bacterium?

1. 1
2. 100
3. 1 thousand
4. 1 million (10^6)
5. 1 billion (10^9)

All answers count (for now)!

Scale and Dimensions of a Bacterial Cell

- A typical bacterium found in the human gut: *Escherichia coli*



- Approximate this as a cylinder

- Volume of cylinder = $L \times A$

  L = length of cylinder

  A = area of cap = $\pi \times R^2$

Volume of Cylinder Representing a Bacterium

- Using units of μm:

$$V = L \times A = L \times \pi \times R^2$$
$$= 2\,\mu\mathrm{m} \times \pi \times (0.5\,\mu\mathrm{m})^2$$
$$= 1.6\,\mu\mathrm{m}^3$$

- Convert to m^3

$$1\,\mu\mathrm{m} = 10^{-6}\,\mathrm{m}$$
$$1.6\,\mu\mathrm{m}^3 \times \left(10^{-6}\,\mathrm{m}/\mu\mathrm{m}\right)^3 = 1.6\times10^{-18}\,\mathrm{m}^3$$

- What about liters or milliliters?

From Cubic Meters to Liters

- An easy-to-remember factoid: $1\,\mathrm{cm}^3$ (“cc”) $= 1\,\mathrm{mL} = 10^{-3}\,\mathrm{L}$
- And, $1\,\mathrm{cm} = 10^{-2}\,\mathrm{m}$

$$1\,\mathrm{cm}^3 \times \left(10^{-2}\,\mathrm{m/cm}\right)^3 = 10^{-6}\,\mathrm{m}^3$$

$$1\,\mathrm{mL} = 10^{-6}\,\mathrm{m}^3$$

$$1\,\mathrm{L} = 10^3\,\mathrm{mL} \times 10^{-6}\,\mathrm{m}^3/\mathrm{mL} = 10^{-3}\,\mathrm{m}^3$$

- For our bacterium:

$$1.6\times10^{-18}\,\mathrm{m}^3 \times 10^3\,\mathrm{L/m}^3 = \boxed{1.6\times10^{-15}\,\mathrm{L}}$$

$$1.6\times10^{-15}\,\mathrm{L} \times 10^3\,\mathrm{mL/L} = 1.6\times10^{-12}\,\mathrm{mL}$$

- How many bacteria would fit into 1 mL? In one 1 μL?

Units of Concentration

- Most convenient: amount of solute per volume of solution
  - g/L (= mg/mL): 1 g solute in 1 L final volume of solution
  - molar (M) = mole/L: 1 mole of solute in 1 L final volume of solution

    1 mole = amount of a substance containing the number of atoms or molecules equal to the number of atoms in 12 g of ^{12}C.

    Number of atoms or molecules in 1 mole of a substance is called Avogadro's number, $N_A \approx 6.02 \times 10^{23}$
- Some less convenient (for purposes of calculation) units of concentration
  - molal: 1 mole of solute dissolved in 1 kg solvent
  - 1%(m/v): 1 g solute in 100 mL final volume of solution
  - 1%(v/v): 1 mL pure liquid in 100 mL final volume of solution

A Source of Confusion: Units for "Molecular Weight"

- Molecular weight or molecular mass:
  - The mass of a single molecule
  - Units: atomic mass unit (u or amu) *or* dalton (Da) or kilodalton (kDa)

    1 amu = 1 Da = mass of one atom of $^{12}C \div 12$
  - Units are often not included, because it is a relative mass, M_r.
  - amu is commonly used in mass spectrometry
  - Da and kDa are very commonly used in biochemistry and molecular biology, especially for proteins and other macromolecules.
- Molar mass:
  - Mass of one mole of a compound
  - Units: g/mol (which doesn't completely make sense)
- Molecular mass of 100 Da $\longrightarrow$ molar mass of 100 g/mol

To Calculate the Amount of Solute in a Solution

- The number of grams in 53 mL of a 5 g/L solution:

$$53\,\text{mL} \times 0.001\,\text{L/mL} = 0.053\,\text{L}$$
$$0.053\,\text{L} \times 5\,\text{g/L} = 0.26\,\text{g}$$

- The number of moles in 1.3 L of a 15 mM solution ($1\,\text{mM} = 0.001\,\text{M}$):

$$15\,\text{mM} \times 0.001\,\text{M/mM} = 0.015\,\text{M} = 0.015\,\text{mol/L}$$
$$1.3\,\text{L} \times 0.015\,\text{mol/L} = 0.0195\,\text{mol}$$

- The number of molecules in 1.3 L of a 15 mM solution:

$$1\,\text{mol} = 6.02{\times}10^{23}\,\text{molecules}$$
$$0.0195\,\text{mol} \times 6.02{\times}10^{23}\,\text{molecules/mol} = 1.17{\times}10^{22}\,\text{molecules}$$

Clicker Question #2

How many moles of water molecules ($M_r = 18$) are in 1 L?

1. ~ 10
2. ~ 30
3. ~ 50 (55)
4. ~ 70

A Special Measure of Concentration for Hydrogen Ions

- Dissociation of water

$$H_2O \rightleftharpoons H^+ + OH^-$$

- Hydrogen ion concentration expressed as pH

$$\mathrm{pH} = -\log\,[H^+]$$

  with $[H^+]$ expressed in molar units

- To convert from pH to molar concentration:

$$[H^+] = 10^{-\mathrm{pH}}\,\mathrm{M}$$

- In a neutral solution, $[H^+] = [OH^-]$

  This happens when $\mathrm{pH} = 7$.

Clicker Question #3

How many moles of hydrogen ions are in a bacterium with an internal pH of 7?

1. $\sim 10^{-22}$
2. $\sim 10^{-17}$
3. $\sim 10^{-12}$
4. $\sim 10^{-7}$

How Many H^+ Ions Are There in a Bacterium?

- Volume = 1.6×10^{-12} L
- $[H^+] = 10^{-pH} M = 10^{-7}$ M
- Moles of H^+:

$$1.6 \times 10^{-15}\ \mathrm{L} \times 10^{-7}\ \mathrm{mol/L} = 1.6 \times 10^{-22}\ \mathrm{moles}$$

- Number of ions:

$$1.6 \times 10^{-22}\ \mathrm{moles} \times 6.02 \times 10^{23}\ \mathrm{ions/mol} \approx 100\ \mathrm{H^+ ions}$$

- Some bacteria grow at pH 9. How many hydrogen ions are in one of these bacteria?