Physical Principles in Biology
Biology 3550
Fall 2017

Lecture 32

Quiz 3 Review and More on Lipids and Membranes

Wednesday, 15 November

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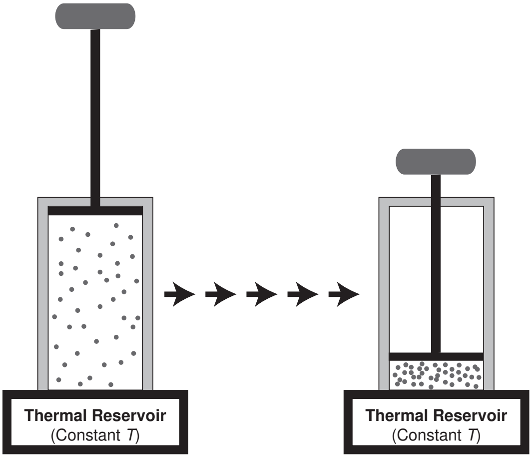
Quiz 3: Problem 1b

- Compression of a nearly ideal gas (N_2)
    - 0.25 moles
    - 0.5 L at start.
    - 0.05 L at end.
    - 25˚C at beginning and end.
- What is the minimum amount of energy required to compress the gas?

Path of Minimum Work: Reversible Isothermal Compression



- Infinitesimally small steps, with temperature kept constant.
- Work for reversible process:

$$w_{\text{rev}} = -nRT \ln \frac{V_2}{V_1}$$

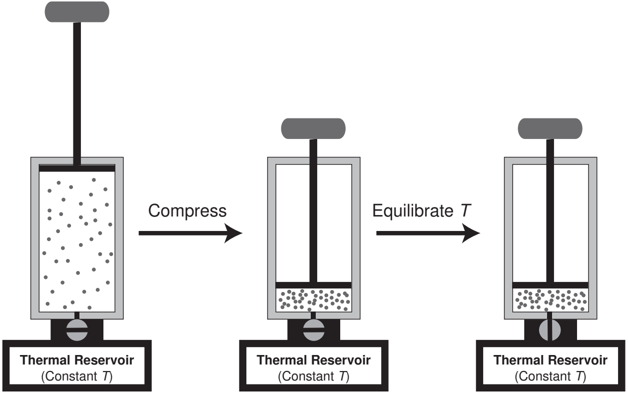
- Heat for reversible process: $(\Delta E = 0)$

$$q_{\text{rev}} = -w_{\text{rev}} = nRT \ln \frac{V_2}{V_1}$$

- Entropy change for system:

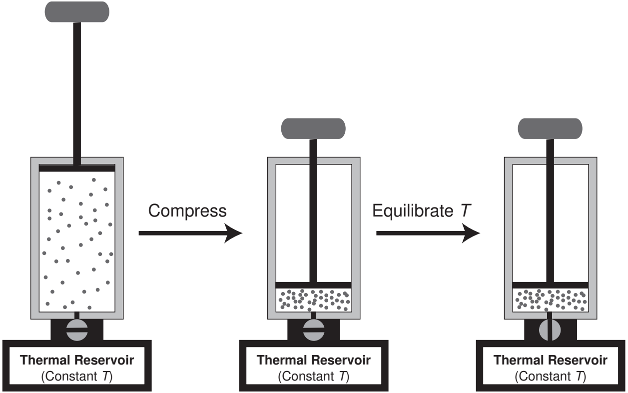
$$\Delta S_{\text{sys}} = \frac{q_{\text{rev}}}{T} = nR \ln \frac{V_2}{V_1}$$

Quiz 3: Problem 1c: Compression via a Different Path



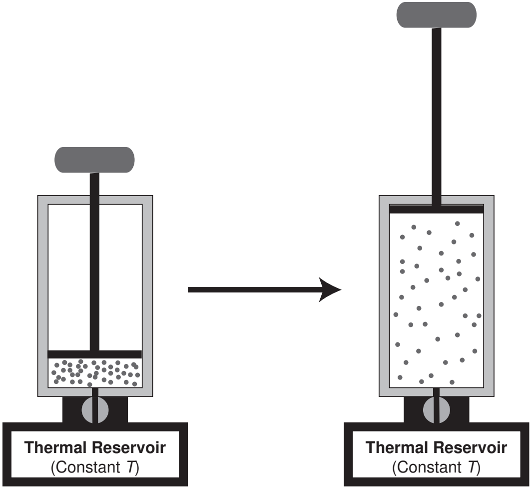
- Gas is compressed adiabatically and *then* allowed to return to 25˚C.
- Work required is $4 \times w_{rev}$
- Why?

Quiz 3: Problem 1d: Compression via a Different Path



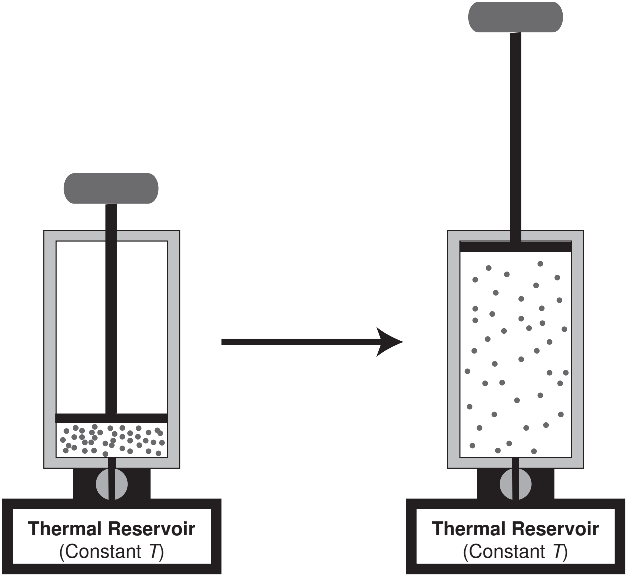
- How can this be done (in a real process) with less work?

Quiz 3: Problem 2: Gas Expansion



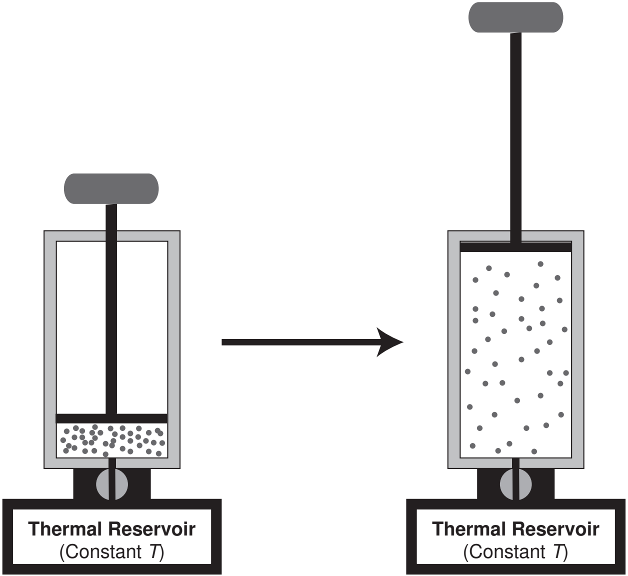
- A real process, *not* reversible.
- Heat is allowed to flow in or out.
- Initial and final temperatures are 25°C.
- $q = 1,000$ J
- Calculate (or infer):
  - ΔE
  - w
  - ΔS_{sys}
  - ΔS_{surr}
  - ΔS_{univ}
  - ΔF

Quiz 3: Problem 2: Gas Expansion



- $\Delta E = ?$
- Temperature at beginning and end are equal.
- $\Delta E = 0$.
  (an inference!)

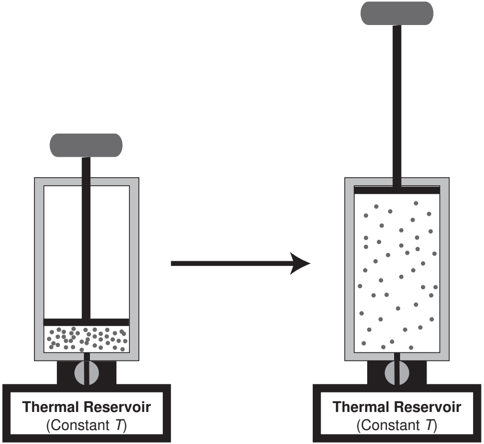
Quiz 3: Problem 2: Gas Expansion



- $w = ?$
- $\Delta E = q + w = 0$

$$\begin{aligned} w &= \Delta E - q \\ &= -q \\ &= -1,000\ \text{J} \end{aligned}$$

Quiz 3: Problem 2: Gas Expansion



- $\Delta S_{sys} = ?$
- $\Delta S_{sys} = q_{rev}/T$
- From problem 1b, for compression:

$$w_{rev} = 1.4\times10^{3}\ \mathrm{J}$$

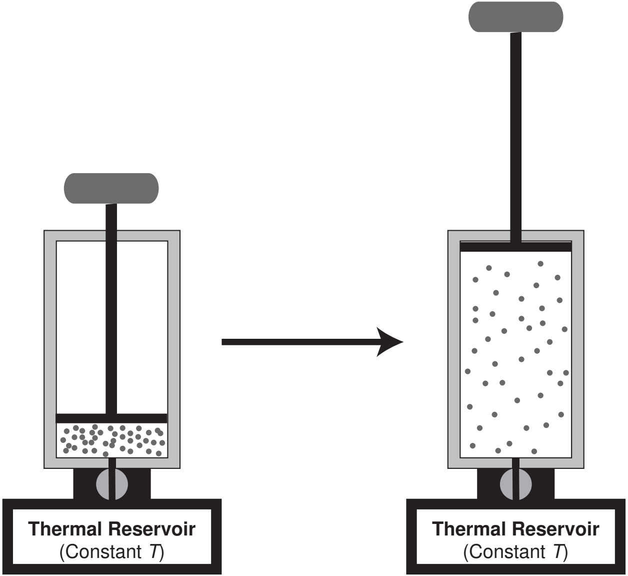
- For the reverse process, expansion:

$$w_{rev} = -1.4\times10^{3}\ \mathrm{J}$$
$$q_{rev} = \Delta E - w_{rev}$$
$$q_{rev} = 1.4\times10^{3}\ \mathrm{J}$$

- The entropy change:

$$\Delta S_{sys} = \frac{q_{rev}}{T} = \frac{1.4\times10^{3}\ \mathrm{J}}{298\ \mathrm{K}}$$
$$= 4.7\ \mathrm{J/K}$$

Quiz 3: Problem 2: Gas Expansion

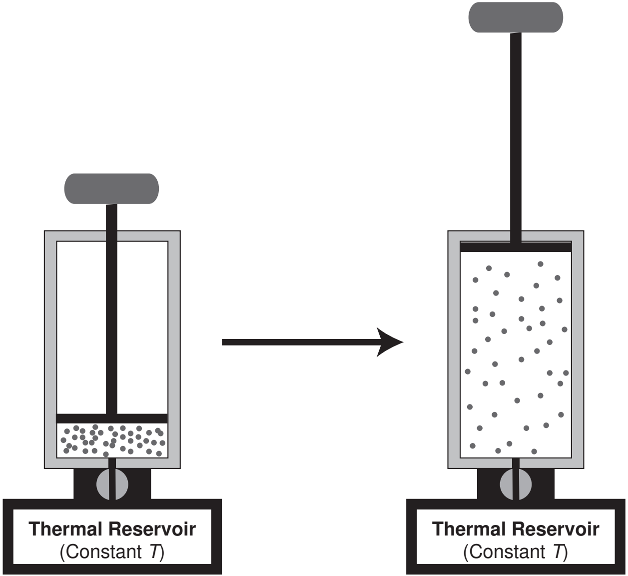


- $\Delta S_{surr} = ?$
- $\Delta S_{surr} = -q/T$

$$\Delta S_{surr} = -\frac{1,000\ \text{J}}{298\ \text{K}}$$

$$= -3.4\ \text{J/K}$$

Quiz 3: Problem 2: Gas Expansion

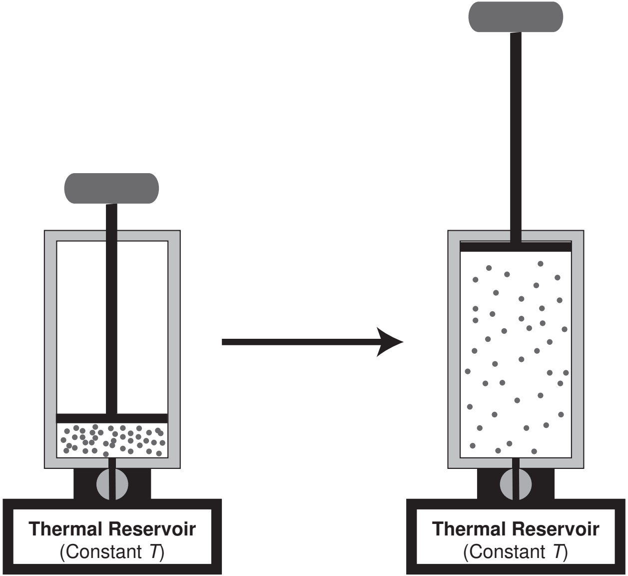


- $\Delta S_{\text{univ}} = ?$
- $\Delta S_{\text{univ}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}}$

$$\Delta S_{\text{univ}} = 4.7\,\text{J/K} - 3.4\,\text{J/K}$$
$$= 1.3\,\text{J/K}$$

- The gas will expand spontaneously!

Quiz 3: Problem 2: Gas Expansion



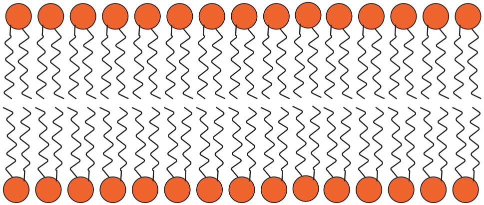
- $\Delta F = ?$
- $\Delta F = \Delta E - T\Delta S_{sys}$

$$\Delta F = 0 - 298\,\text{K} \times 4.7\,\text{J/K}$$
$$= -1.4\times10^{3}\,\text{J}$$

- Or:

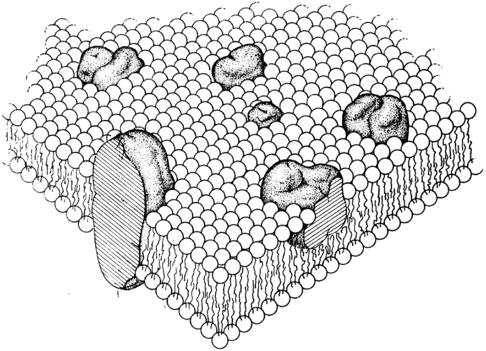
$$\Delta F = w_{rev}$$
$$= -1.4\times10^{3}\,\text{J}$$

Phospholipids and Bilayers



- Large two dimensional sheets.
- Non-polar tails sequestered away from water.
- Polar head groups exposed to water on each side.
- Why don't phospholipids form micelles, like detergents do?

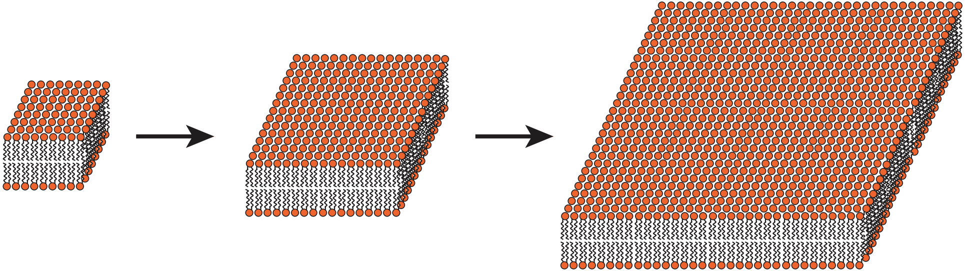
General Structure of Biological Membranes



- Proteins embedded in bilayer control passage of other molecules.
- Many other functions of membrane proteins!

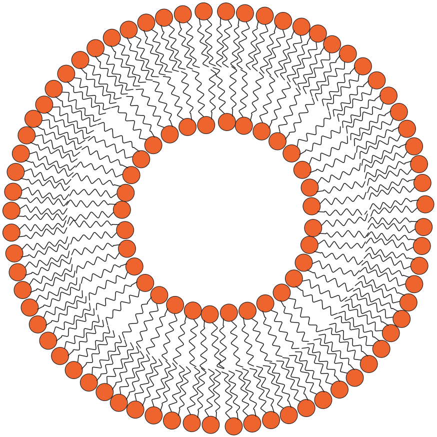
Singer, S. & Nicolson, G. (1972). The fluid mosaic model of the structure of cell membranes. *Science*, 175, 720–731. http://dx.doi.org/10.1126/science.175.4023.720

Growth of Bilayer Sheets



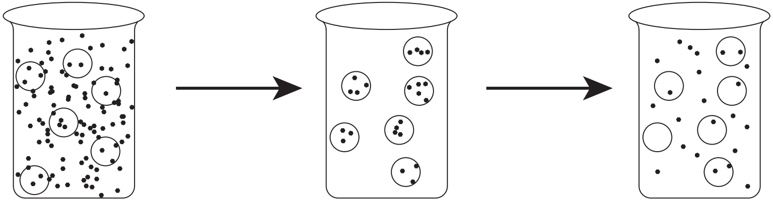
- As sheet grows, surface area of non-polar lipids exposed at edges grows.
- Hydrophobic effect counteracts resistance of the sheet to curve.

Bilayer Sheets Form Vesicles



- Bilayers close on themselves to eliminate edges.
- 50 nm–1 μM in diameter.
- Function in vivo to store and transport specific molecules (*e.g.*, neurotransmitters)
- Formed experimentally by vigorously mixing bilayers in water.

Using Vesicles to Measure Permeability of Bilayers



- Form vesicles in presence of molecules of interest.
- Separate vesicles from external molecules.
- Allow molecules to diffuse across bilayers.
- Separate vesicles from external molecules and measure concentrations.
- What determines rate of escape?
  Fick's first law!

Diffusion Across Vesicle Bilayer

- Fick's first law: $J = -D\dfrac{dC}{dx}$

- Concentration gradient: $\dfrac{dC}{dx} \approx \dfrac{C_{\text{in}} - C_{\text{out}}}{\text{Bilayer thickness}}$

- A new parameter commonly used in this and other contexts: permeability coefficient, P (not to be confused with pressure!).

  Represents combination of diffusion coefficient and membrane thickness:

  $$P = \frac{D}{\Delta x}$$

  Units:

  $$\text{m}^2/\text{s} \div \text{m} = \text{m}/\text{s}$$

Fick's First Law Expressed Using the Permeation Coefficient

$$J = -D\frac{dC}{dx} \approx -D\frac{\Delta C}{\Delta x} = -P\Delta C$$

- In practice: Measure flux and calculate P

$$P = -\frac{J}{\Delta C}$$

- What do we need to know?
  - Concentration of molecules inside vesicle and out.
  - Rate of molecules diffusing (moles/time).
  - Surface area of vesicles (J has units of $\mathrm{mol} \cdot \mathrm{m}^{-2}\mathrm{s}^{-1}$).
  - Don't need to know membrane thickness!
- P reflects both molecule and bilayer (or other kind of membrane)

Clicker Question #1

What kind of ions or molecules would you expect to have the **largest** permeability coefficients for phospholipid bilayers?

1. Sugars
2. Amino acids
3. Water
4. Nucleotides
5. Small ions like Na^+, K^+ or Cl^-

All answers count for now!

Clicker Question #2

What kind of ions or molecules would you expect to have the **smallest** permeability coefficients for phospholipid bilayers?

1. Sugars
2. Amino acids
3. Water
4. Nucleotides
5. Small ions like Na^+, K^+ or Cl^-

All answers count for now!