Physical Principles in Biology
Biology 3550
Fall 2017

Lecture 3:

More on Measurements and Units

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Clicker Question #1

Which of the following are "basic" units in the International System (SI)?

(Choose up to two.)

1. Smoot
2. kilogram
3. meter
4. Liter
5. newton

Basic Dimensions in the Current Metric System

International System of Units (SI)

Dimension	Symbol	SI Unit
Length	L	meter (m)
Mass	M	kilogram (kg)
Time	t	second (s)
Thermodynamic temperature	T	kelvin (K)
Electric current	I	ampere (A)
Amount of substance	?	mole (mol)
Luminous intensity	I_v	candela (cd)

- Choice of some of the basic units is somewhat arbitrary.

Examples of Derived SI Units

Dimension	Symbol	SI Unit
Area	$A = L^2$	m^2
Volume	$V = L^3$	m^3
Velocity	$v = L/t$	m/s
Acceleration	$a = v/t = L/t^2$	m/s^2
Force	$f = ma = m \cdot L/t^2$	newton (N) = $kg{\cdot}m/s^2$
Energy	$E = f \cdot L$	J = Nm = $kg{\cdot}m^2/s^2$

Standard Prefixes for SI Units

prefix	abbreviation	multiplier	examples
nano	n	10^{-9}	nm, ng
micro	μ	10^{-6}	μm, μg
milli	m	10^{-3}	mm, mg
centi	c	10^{-2}	cm, cg
deci	d	10^{-1}	dm, dg
kilo	k	10^{3}	km, kg
mega	M	10^{6}	Mm, Mg

You should know these!

Using Units in Calculations

- Dimensional analysis: Treat dimensions and units as symbols that can be manipulated by standard rules of algebra.
- Conversion factors:
  - As a rule: “To convert from kilometers to meters, multiply by 1,000.”
  - As an equation: $1\,\text{km} = 1,000\,\text{m}$
- Which of these equations make sense:
  - $1\,\text{mg} = 1\,\text{g}/1,000$
  - $1\,\text{mile} = 1.609344\,\text{km}$
  - $1\,\text{kg} = 1\,\text{liter}$
  - $1\,\text{meter}/\text{s} = 1,1969\,\text{feet}/\text{hour}$

Algebraic Rearrangement of a Conversion Factor

$$1\,\text{km} = 1,000\,\text{m}$$

$$\frac{1\,\text{km}}{1,000\,\text{m}} = 1$$

$$\frac{1,000\,\text{m}}{1\,\text{km}} = 1$$

$$1\,\text{m} = 0.001\,\text{km}$$

- All of these equations are valid and equivalent.
- Any quantity can be multiplied (or divided) by 1 to give the same value.

Conversion by Multiplication

- Convert 37 miles to kilometers

$$1\ \text{mi} = 1.609344\ \text{km}$$
$$1.609344\ \text{km}/\text{mi} = 1$$

$$\begin{aligned} 37\ \text{mi} \times 1.609344\ \text{km}/\text{mi} &= 59.6\ \text{mi} \cdot \text{km}/\text{mi} \\ &= 59.6\ \text{km} \end{aligned}$$

- What if we divide instead of multiply?

$$37\ \text{mi} \div 1.609344\ \text{km}/\text{mi} = 23\ \text{mi}^2/\text{km}$$

This is correct algebraically, but it doesn't make much sense physically!

Clicker Question #2

"English" units for mass:

- 1 lb = 16 oz (avoirdupois)
- 1 oz = 28.349523125 g
- 1 stone = 14 Lb

If someone weighs 11 stone, what is that person's mass in kg?

1. ~ 50 kg
2. ~ 70 kg
3. ~ 90 kg
4. ~ 110 kg

Stones to kg

$$11\,\text{stone} \times 14\,\text{lb/stone} = 154\,\text{lb}$$

$$154\,\text{lb} \times 16\,\text{oz/lb} = 2.46{\times}10^3\,\text{oz}$$

$$2.46{\times}10^3\,\text{oz} \times 28.35\,\text{g/oz} = 6.98{\times}10^4\,\text{g}$$

$$6.98{\times}10^4\,\text{g} \div 1000\,\text{g/kg} \approx 70\,\text{kg}$$

$$1\,\text{kg} \approx 2.2\,\text{lb}$$

Clicker Question #3

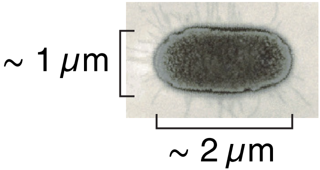
How many hydrogen ions (H^+) are in a typical bacterium?

1. 1
2. 100
3. 1 thousand
4. 1 million (10^6)
5. 1 billion (10^9)

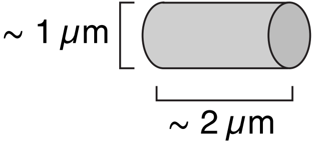
All answers count (for now)!

Scale and Dimensions of a Bacterial Cell

- A typical bacterium found in the human gut: *Escherichia coli*



- Approximate this as a cylinder



- Volume of cylinder = $L \times A$

  L = length of cylinder

  A = area of cap = $\pi \times R^2$

Volume of Cylinder Representing a Bacterium

- Using units of μm:

$$\begin{aligned} V = L \times A &= L \times \pi \times R^2 \\ &= 2\,\mu\text{m} \times \pi \times (0.5\,\mu\text{m})^2 \\ &= 1.6\,\mu\text{m}^3 \end{aligned}$$

- Convert to m^3

$$1\,\mu\text{m} = 10^{-6}\,\text{m}$$

$$1.6\,\mu\text{m}^3 \times \left(10^{-6}\,\text{m}/\mu\text{m}\right)^3 = 1.6{\times}10^{-18}\,\text{m}^3$$

- What about liters or milliliters?

From Cubic Meters to Liters

- An easy-to-remember factoid: $1\,\text{cm}^3$ (“cc”) $= 1\,\text{mL} = 10^{-3}\,\text{L}$
- And, $1\,\text{cm} = 10^{-2}\,\text{m}$

$$1\,\text{cm}^3 \times \left(10^{-2}\,\text{m}/\text{cm}\right)^3 = 10^{-6}\,\text{m}^3$$

$$1\,\text{mL} = 10^{-6}\,\text{m}^3$$

$$1\,\text{L} = 10^3\,\text{mL} \times 10^{-6}\,\text{m}^3/\text{mL} = 10^{-3}\,\text{m}^3$$

- For our bacterium:

$$1.6{\times}10^{-18}\,\text{m}^3 \times 10^3\,\text{L}/\text{m}^3 = 1.6{\times}10^{-15}\,\text{L}$$

$$1.6{\times}10^{-15}\,\text{L} \times 10^3\,\text{mL}/\text{L} = 1.6{\times}10^{-12}\,\text{mL}$$

- How many bacteria would fit into 1 mL? In one 1 μL?

Units of Concentration

- Most convenient: amount of solute per volume of solution
  - g/L (= mg/mL): 1 g solute in 1 L final volume of solution
  - molar (M) = mole/L: 1 mole of solute in 1 L final volume of solution

    1 mole = amount of a substance containing the number of atoms or molecules equal to the number of atoms in 12 g of ^{12}C.

    Number of atoms or molecules in 1 mole of a substance is called Avogadro's number, $N_A \approx 6.02 \times 10^{23}$
- Some less convenient (for purposes of calculation) units of concentration
  - molal: 1 mole of solute dissolved in 1 kg solvent
  - 1%(m/v): 1 g solute in 100 mL final volume of solution
  - 1%(v/v): 1 mL pure liquid in 100 mL final volume of solution

A Source of Confusion: Units for “Molecular Weight”

- Molecular weight or molecular mass:
  - The mass of a single molecule
  - Units: atomic mass unit (u or amu) *or* dalton (Da) or kilodalton (kDa)

    1 amu = 1 Da = mass of one atom of ^{12}C $\div 12$
  - Units are often not included, because it is really a relative mass, M_r.
  - amu is commonly used in mass spectrometry
  - Da and kDa are very commonly used in biochemistry and molecular biology, especially for proteins and other macromolecules.
- Molar mass:
  - Mass of one mole of a compound
  - Units: g/mol
- Molecular mass of 100 Da $\longrightarrow$ molar mass of 100 g/mol

To Calculate the Amount of Solute in a Solution

- The number of grams in 53 mL of a 5 g/L solution:

$$53\,\text{mL} \times 0.001\,\text{L/mL} = 0.053\,\text{L}$$
$$0.053\,\text{L} \times 5\,\text{g/L} = 0.26\,\text{g}$$

- The number of moles in 1.3 L of a 15 mM solution ($1\,\text{mM} = 0.001\,\text{M}$):

$$15\,\text{mM} \times 0.001\,\text{M/mM} = 0.015\,\text{M} = 0.015\,\text{mol/L}$$
$$1.3\,\text{L} \times 0.015\,\text{mol/L} = 0.0195\,\text{mol}$$

- The number of molecules in 1.3 L of a 15 mM solution:

$$1\,\text{mol} = 6.02\times10^{23}\,\text{molecules}$$
$$0.0195\,\text{mol} \times 6.02\times10^{23}\,\text{molecules/mol} = 1.17\times10^{22}\,\text{molecules}$$

Clicker Question #4

How many moles of water molecules ($M_r = 18$) are in 1 L?

1. ~ 10
2. ~ 30
3. ~ 50
4. ~ 70