

Physical Principles in Biology  
Biology 3550  
Fall 2017

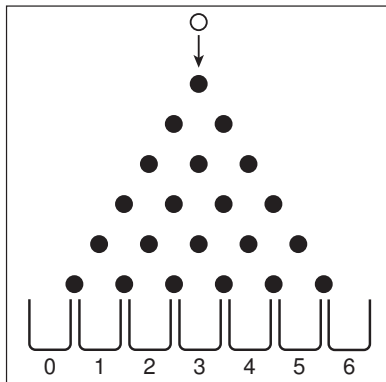
## Lecture 7

### Plinko Probabilities - Part 2

Wednesday, 6 September

©David P. Goldenberg  
University of Utah  
goldenberg@biology.utah.edu

# A Six-row Plinko

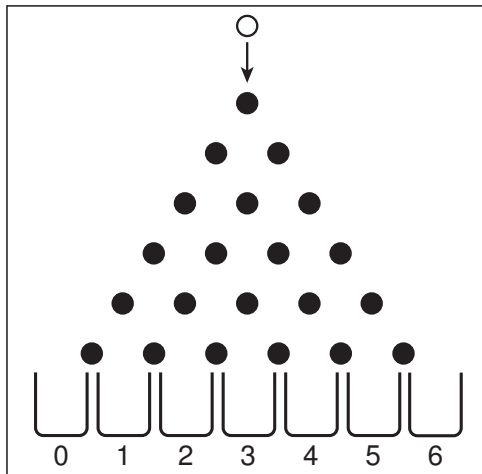


- For  $n$  plinko rows, there will be  $n + 1$  buckets for balls to land in.
- For convenience, buckets are numbered from 0 to  $n$ .

## Defining the Sample Set and Events for the Plinko

- The sample set contains all of the possible outcomes, which we define as all of the possible paths.
  - For an  $n$ -row plinko, there are  $2^n$  paths.
  - If the ball is equally likely to turn right or left at each peg, then all of the paths are equally probable.
  - For an honest plinko, the probability of each outcome is  $2^{-n}$ .
  - For the 6-row plinko, there are 64 paths, each with a probability of  $1/64$ .
- Define events as final positions of ball, *i.e.*, bucket numbers.
  - Different events have different probabilities because there are different numbers of paths to the different buckets.
  - The challenge is to count the paths to each bucket. The more buckets there are, the harder this is going to be!  
(Unless we can find a simple formula.)

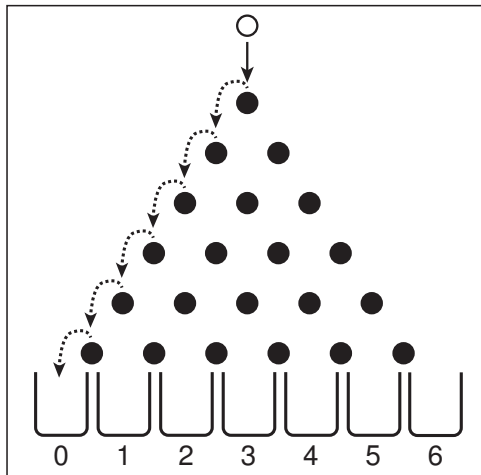
# Need to Count the Number of Paths to Each Final Position



Bucket No.	Paths
0	
1	
2	
3	
4	
5	
6	

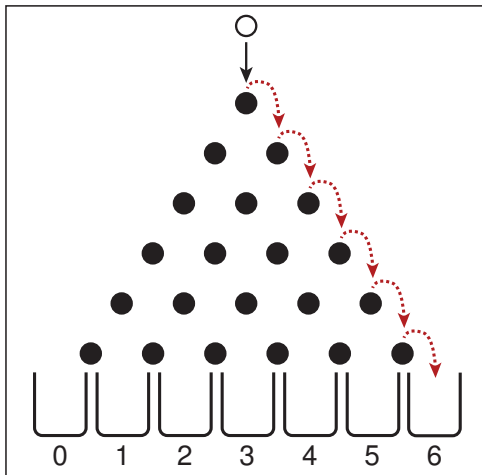
# Clicker Question #1

How many paths are there to bucket 0?



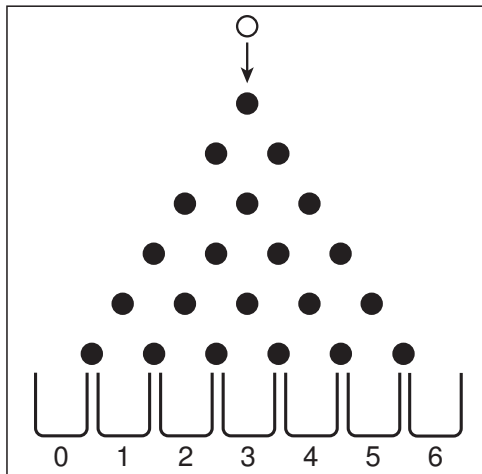
- 1 1
- 2 2
- 3 4
- 4 6
- 5 15
- 6 20

# How Many Paths to Bucket 6?



Bucket No.	Paths
0	1
1	
2	
3	
4	
5	
6	1

# How Many Paths to Bucket 1?

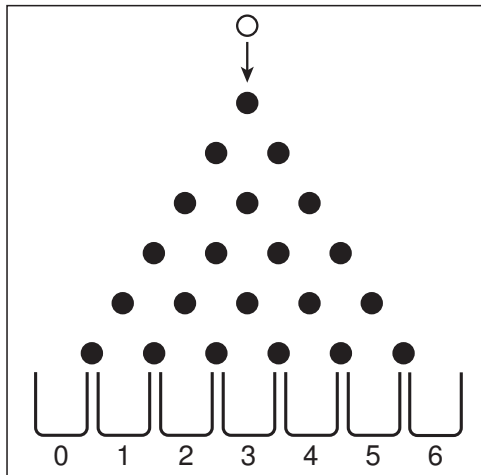


Bucket No.	Paths
0	1
1	
2	
3	
4	
5	
6	1

- Each path to bucket 1 includes 1 turn to the right and 5 to the left.

## Clicker Question #2

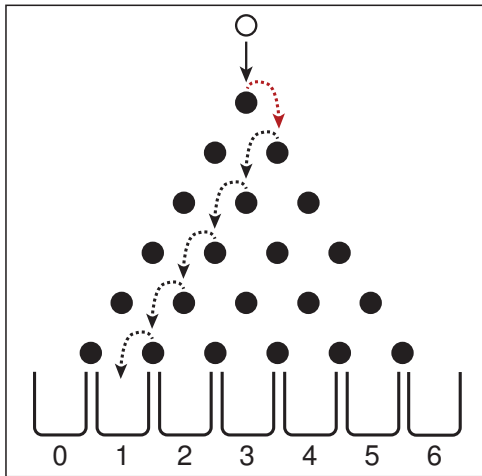
How many paths are there to bucket 1?



- 1 1
- 2 2
- 3 4
- 4 **6**
- 5 15
- 6 20



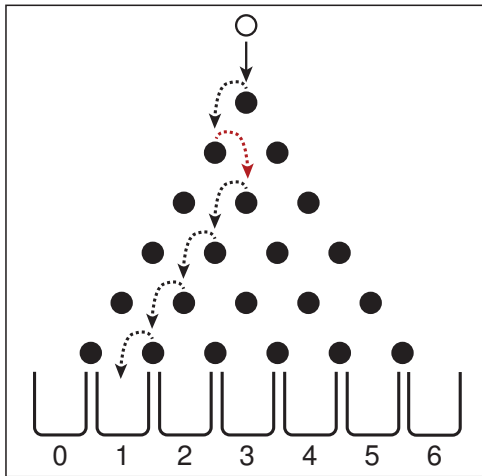
# How Many Paths to Bucket 1?



Bucket No.	Paths
0	1
1	
2	
3	
4	
5	
6	1

- There are six rows at which the turn to the right can occur.

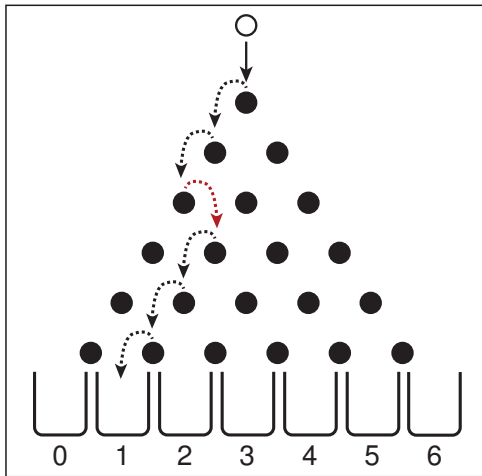
# How Many Paths to Bucket 1?



Bucket No.	Paths
0	1
1	
2	
3	
4	
5	
6	1

- There are six rows at which the turn to the right can occur.

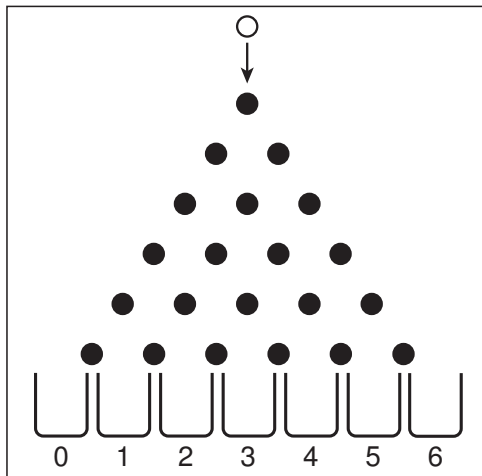
# How Many Paths to Bucket 1?



Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	
6	1

- There are six rows at which the turn to the right can occur.

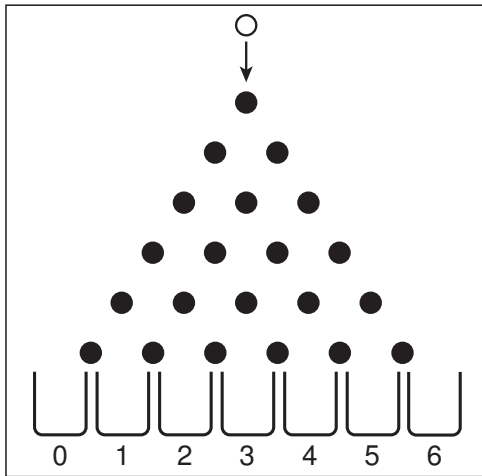
# How Many Paths to Bucket 5?



Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	6
6	1

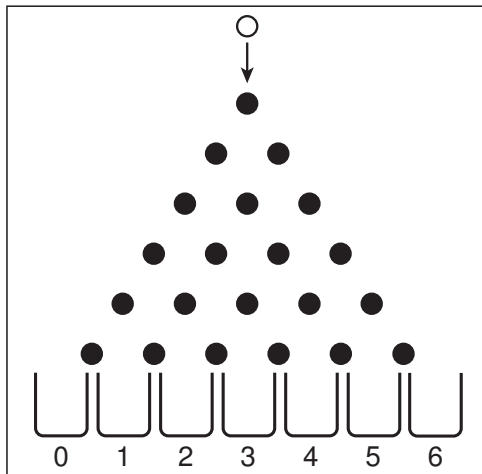
- Each path to bucket 5 includes 5 turns to the right and 1 to the left. There are six rows at which the turn to the left can occur.

# How Many Paths to Bucket 2?



Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	6
6	1

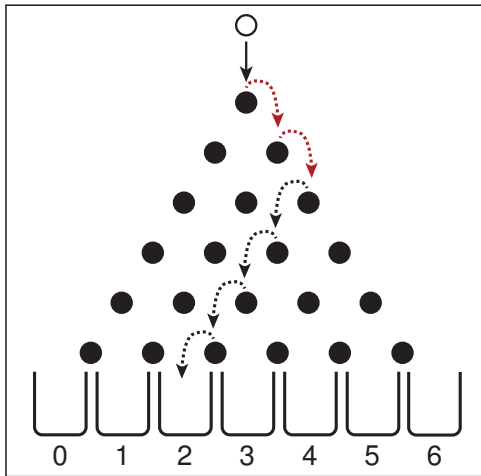
# How Many Paths to Bucket 2?



Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	6
6	1

- Each path to bucket 2 includes 2 turns to the right and 4 to the left.

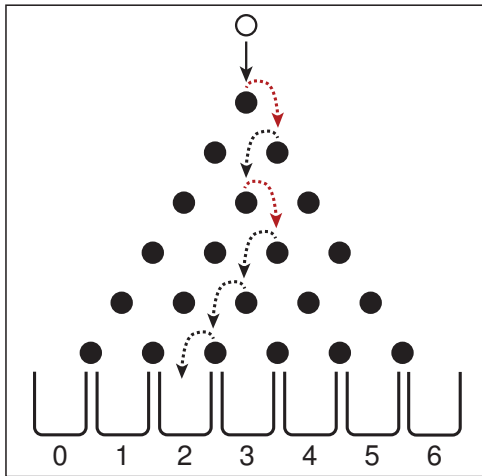
# How Many Paths to Bucket 2?



Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	6
6	1

- Each path to bucket 2 includes 2 turns to the right and 4 to the left.

# How Many Paths to Bucket 2?



Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	6
6	1

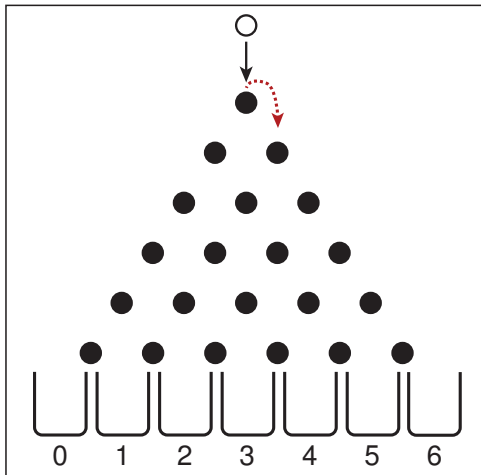
- Each path to bucket 2 includes 2 turns to the right and 4 to the left.





# How Many Paths to Bucket 2?

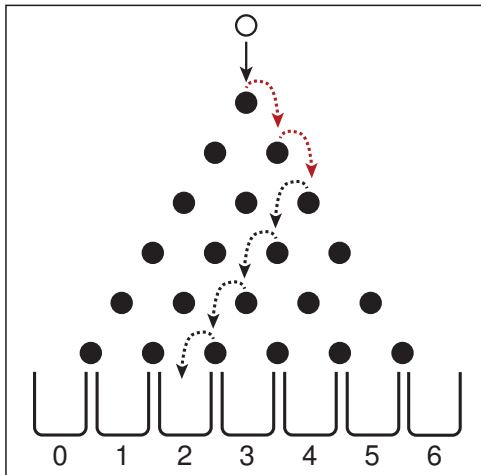
If the first turn is to the right, how many paths are there to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	
2	
3	
4	
5	
6	

# How Many Paths to Bucket 2?

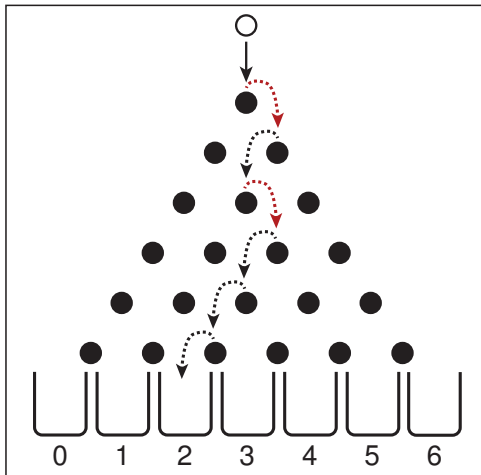
If the first turn is to the right, how many paths are there to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	
2	
3	
4	
5	
6	

# How Many Paths to Bucket 2?

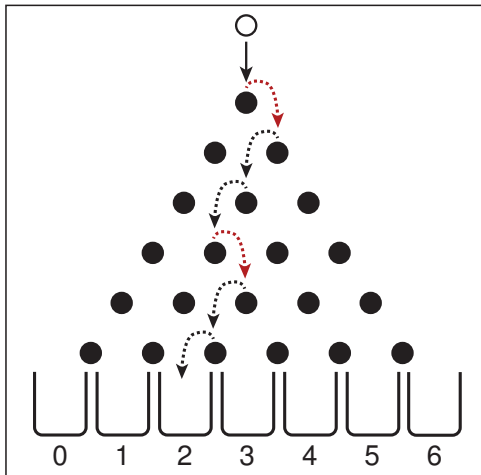
If the first turn is to the right, how many paths are there to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	
2	
3	
4	
5	
6	

# How Many Paths to Bucket 2?

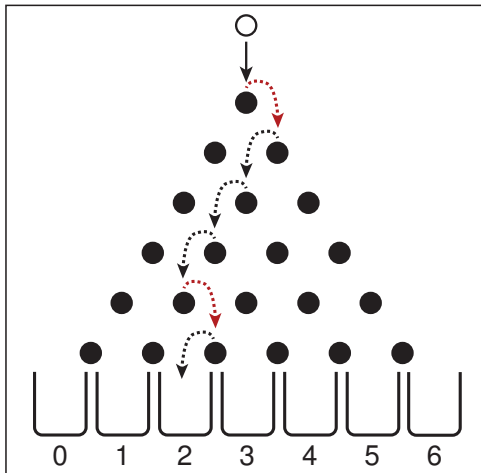
If the first turn is to the right, how many paths are there to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	
2	
3	
4	
5	
6	

# How Many Paths to Bucket 2?

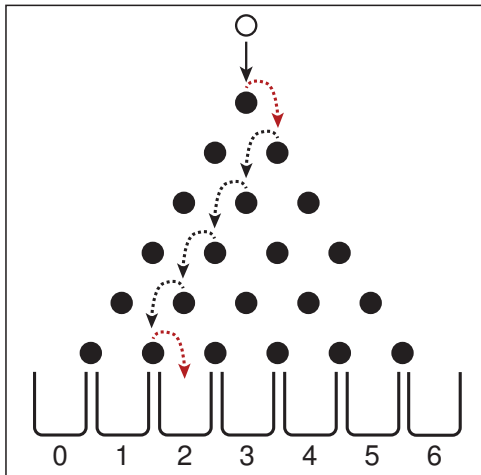
If the first turn is to the right, how many paths are there to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	
2	
3	
4	
5	
6	

# How Many Paths to Bucket 2?

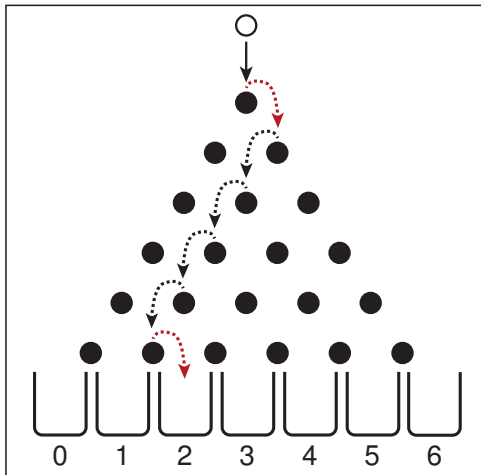
If the first turn is to the right, how many paths are there to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	
2	
3	
4	
5	
6	

# How Many Paths to Bucket 2?

If the first turn is to the right, how many paths are there to bucket 2?

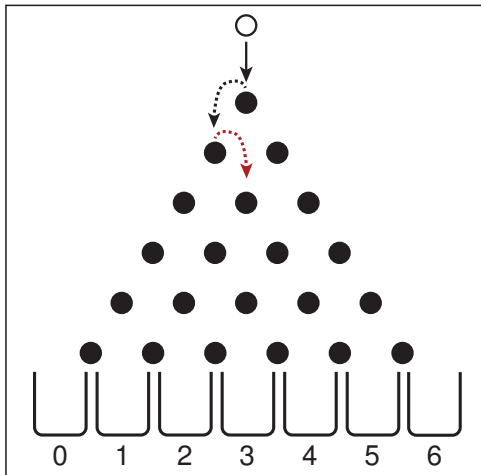


1 <sup>st</sup> right turn row	Paths
1	5
2	
3	
4	
5	
6	



# How Many Paths to Bucket 2?

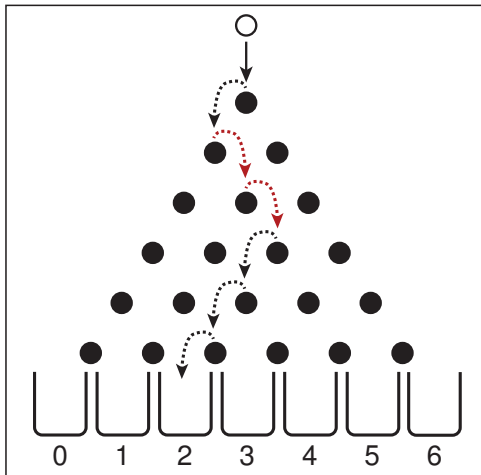
If the first turn to the right is at row 2, how many paths to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	5
2	
3	
4	
5	
6	

# How Many Paths to Bucket 2?

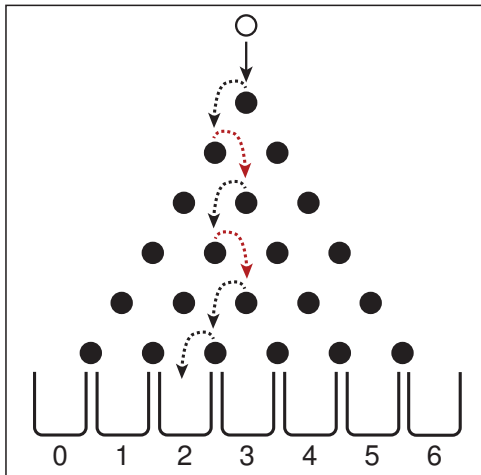
If the first turn to the right is at row 2, how many paths to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	5
2	
3	
4	
5	
6	

# How Many Paths to Bucket 2?

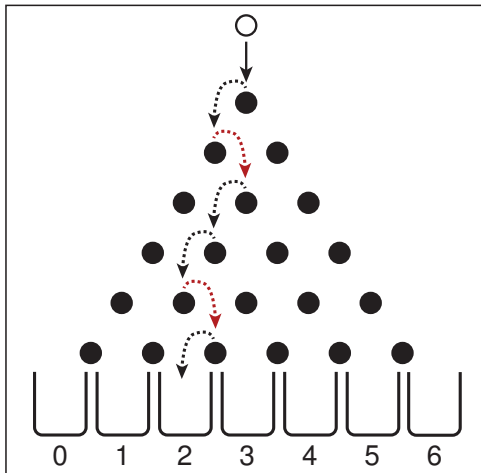
If the first turn to the right is at row 2, how many paths to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	5
2	
3	
4	
5	
6	

# How Many Paths to Bucket 2?

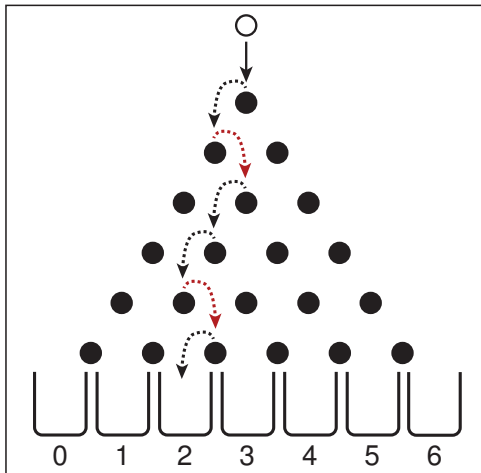
If the first turn to the right is at row 2, how many paths to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	5
2	
3	
4	
5	
6	

# How Many Paths to Bucket 2?

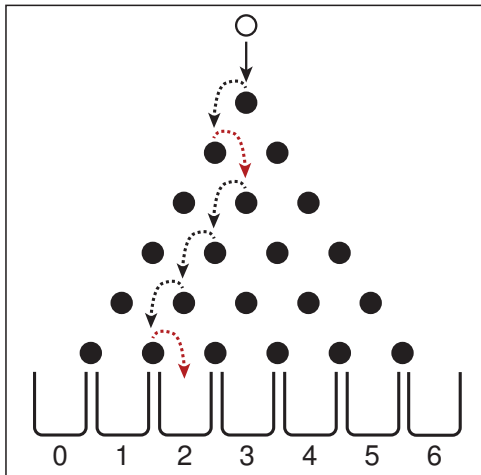
If the first turn to the right is at row 2, how many paths to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	5
2	
3	
4	
5	
6	

# How Many Paths to Bucket 2?

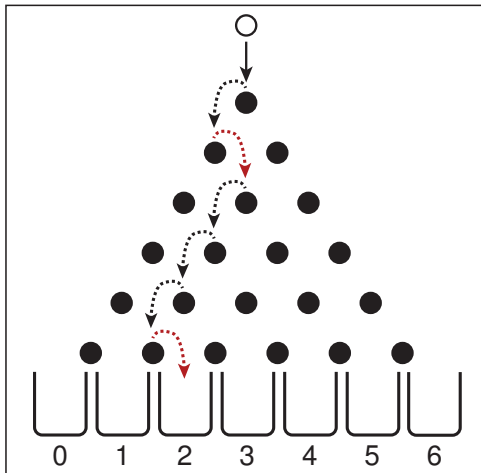
If the first turn to the right is at row 2, how many paths to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	5
2	
3	
4	
5	
6	

# How Many Paths to Bucket 2?

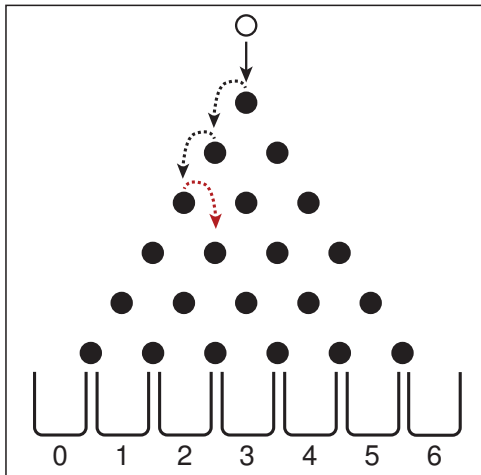
If the first turn to the right is at row 2, how many paths to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	5
2	4
3	
4	
5	
6	

# How Many Paths to Bucket 2?

If the first turn to the right is at row 3, how many paths to bucket 2?

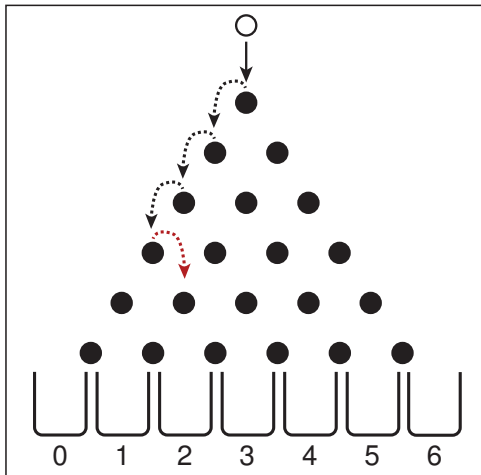


1 <sup>st</sup> right turn row	Paths
1	5
2	4
3	3
4	
5	
6	



# How Many Paths to Bucket 2?

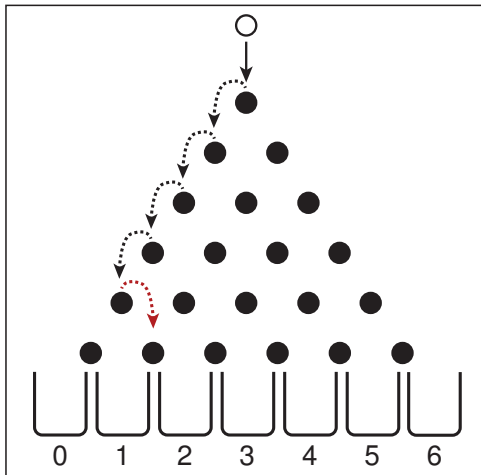
If the first turn to the right is at row 4, how many paths to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	5
2	4
3	3
4	2
5	
6	

# How Many Paths to Bucket 2?

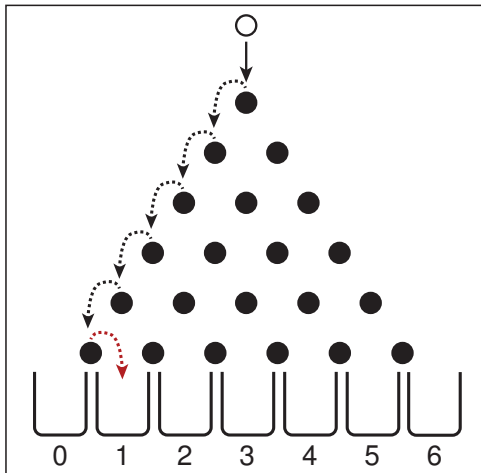
If the first turn to the right is at row 5, how many paths to bucket 2?



1 <sup>st</sup> right turn row	Paths
1	5
2	4
3	3
4	2
5	1
6	

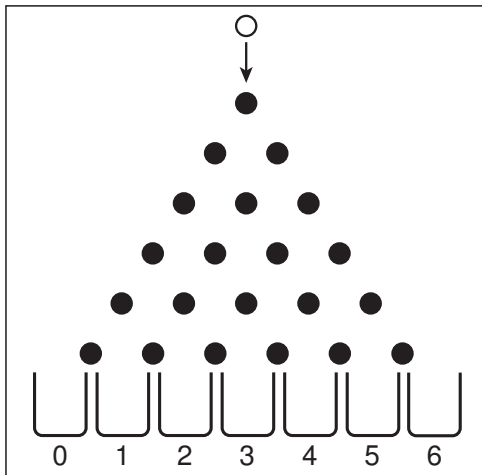
# How Many Paths to Bucket 2?

If the first turn to the right is at row 6, how many paths to bucket 2?



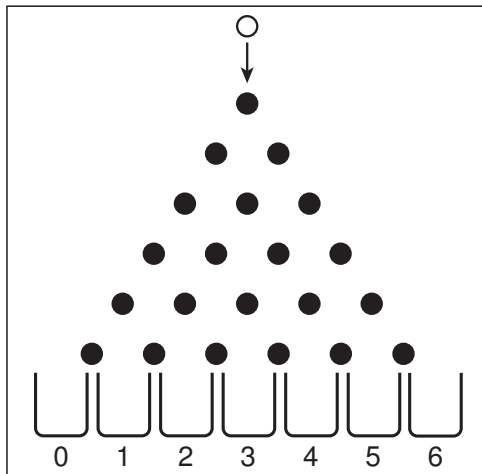
1 <sup>st</sup> right turn row	Paths
1	5
2	4
3	3
4	2
5	1
6	0

# How Many Paths to Bucket 2?



Bucket No.	Paths
0	1
1	6
2	15
3	
4	
5	6
6	1

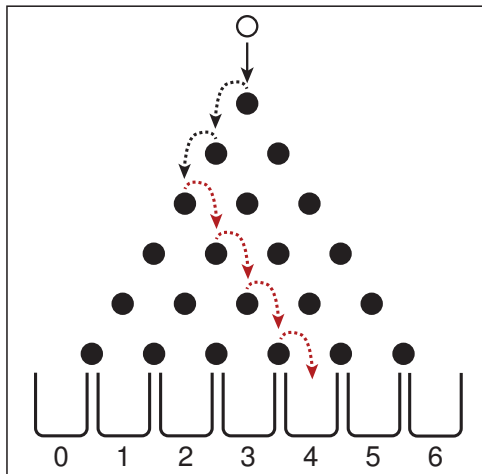
# How Many Paths to Bucket 4?



Bucket No.	Paths
0	1
1	6
2	15
3	
4	
5	6
6	1

- Each path to bucket 4 includes 4 turns to the right and 2 to the left.

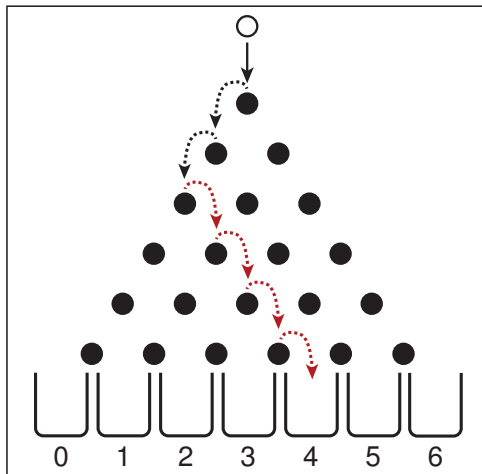
# How Many Paths to Bucket 4?



Bucket No.	Paths
0	1
1	6
2	15
3	
4	
5	6
6	1

- Each path to bucket 4 includes 4 turns to the right and 2 to the left.

# How Many Paths to Bucket 4?

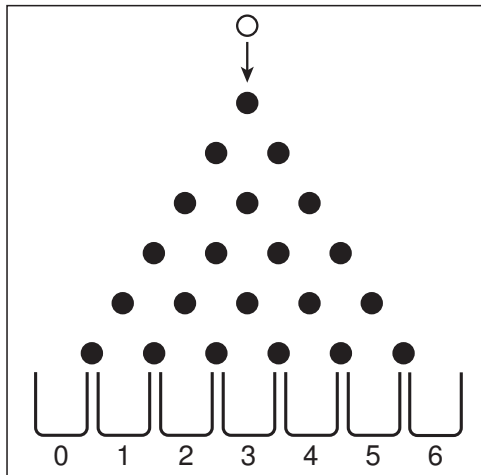


Bucket No.	Paths
0	1
1	6
2	15
3	
4	15
5	6
6	1

- Each path to bucket 4 includes 4 turns to the right and 2 to the left.

# Clicker Question #3

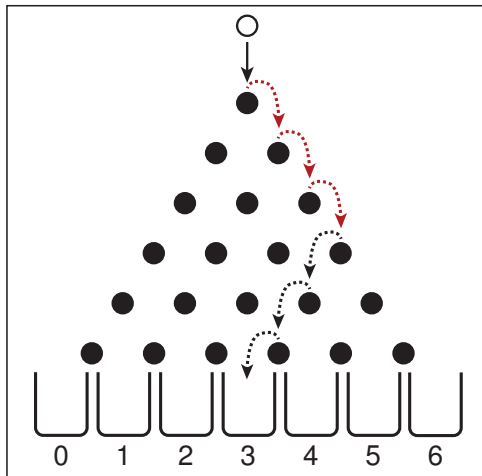
How many paths are there to bucket 3?



- 1 1
- 2 2
- 3 4
- 4 6
- 5 15
- 6 20



# How Many Paths to Bucket 3



Bucket No.	Paths
0	1
1	6
2	15
3	
4	15
5	6
6	1

- Each path to bucket 3 includes 3 turns to the right and 3 to the left.
- Counting the paths to bucket 3 looks hard!

## Another Way to Count the Paths to Bucket 2

- 2 turns to the right and 4 turns to the left.
- There are 6 rows where one turn could be placed.
- There are 5 rows where a second turn could be placed.  
 $6 \times 5 = 30$
- BUT, this assumes that the turns can be placed in either order!
- The first turn has to come before the second, so each possible path has been counted twice.
- The correct count:  $6 \times 5 \times \frac{1}{2} = 15$
- A general strategy: Count all of the possible places where each right turn could be placed, allowing all possible orders, and then correct for over counting.

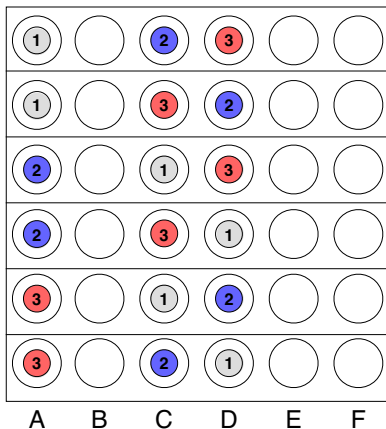
# Counting the Paths to Bucket 3

- 3 turns to the right and 3 to the left.
- Ignoring the order of placement:
  - 6 rows where a first turn can be placed.
  - 5 rows where a second turn can be placed.
  - 4 rows where a third turn can be placed.
  - $6 \times 5 \times 4 = 120$
- But, this assumes turns can be placed in any order!
- How do we determine how many times the paths have been over counted?

# A Related Problem: Placing Beans in Cups

- Suppose that we have 3 beans, each with the number 1, 2 or 3 printed on it.
- How many different ways are there to place the beans in 6 cups? (with no more than one bean per cup)
  - 6 cups where the first bean can be placed.
  - 5 cups where the second bean can be placed.
  - 4 cups where the third bean can be placed.
  - $6 \times 5 \times 4 = 120$
- These are all different, because the beans are distinguishable.
- It is also the number of over-counted paths to bucket 3 in the plinko.

## 3 Labeled Beans in 3 Cups



- How many ways are there to put 3 labeled beans in 3 **specific** cups? (A,C and D)
- $3 \times 2 \times 1 = 6$
- Only one of these has the order 1-2-3.

# Three Labeled Beans in Six Cups

- If all orders are counted:

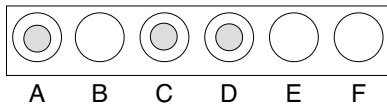
Number of ways to place beans is:  $6 \times 5 \times 4 = 120$

- If only the placements with the order 1-2-3 are counted:

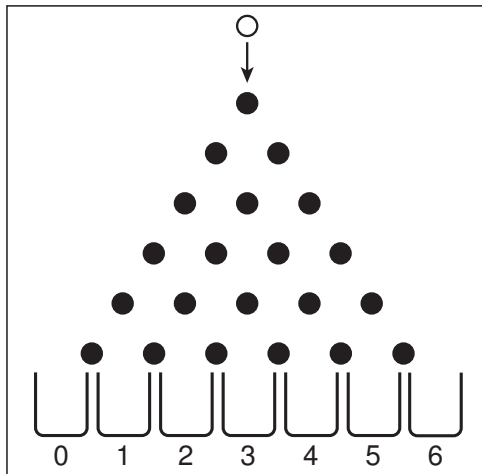
Number of ways to place beans is:  $(6 \times 5 \times 4) \div 6 = 20$

- Also the number of ways to place 3 turns to the right in the 6-row plinko!

- Also the number of ways to place 3 indistinguishable beans in 6 cups.

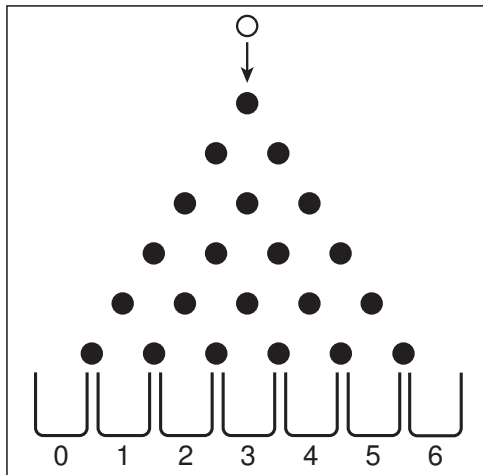


# The Full Path Count for the Six-row Plinko



Bucket No.	Paths
0	1
1	6
2	15
3	20
4	15
5	6
6	1

# Probabilities for the Six-row Plinko



Bucket No.	Paths	Probability
0	1	$1/64 \approx 0.016$
1	6	$6/64 \approx 0.094$
2	15	$15/64 \approx 0.234$
3	20	$20/64 \approx 0.312$
4	15	$15/64 \approx 0.234$
5	6	$6/64 \approx 0.094$
6	1	$1/64 \approx 0.016$