

Biology 3820: Physical Principles in Biology  
Fall Semester - 2016

Problem Set 1

Due: 12:00, midnight, Tuesday, 6 September

Notes:

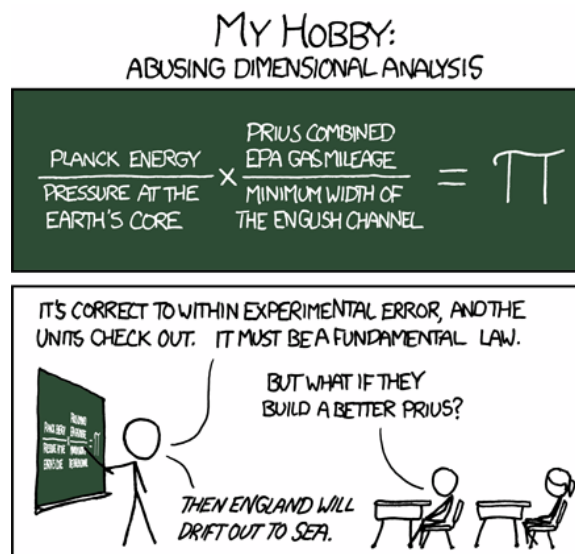
Be sure to show your work, and use the proper units.

You are encouraged to work together on the homework assignments and to use outside resources, including the internet. However, **the work you turn in must be your own!** Any text must be clearly distinguishable from that of other students, and other sources must be properly cited. Text from other sources must be clearly identified by quotation marks. Furthermore, extensive quotations, even with proper citation, will not be considered satisfactory answers to questions. Copying and pasting does not demonstrate mastery of the material!

Submit your work as a pdf file via Canvas. Your work must be typed: Scans of hand-written work will not be graded.

Problems worthy of attack prove their worth by fighting back.  
Piet Hein (or maybe Paul Erdos)

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1. The cartoon below comes from <http://xkcd.com>, described by its author, Randall Munroe, as a “webcomic of romance, sarcasm, math and language.” (Read at your own risk.)



Whether or not you think this is funny, is the equation correct?

- (a) Is the dimensional analysis correct? That is, do the dimensions on the left hand side of the equation all cancel out to leave a dimensionless number? To answer this question, write out the basic dimensions for each term. You can use either generic dimensions ( $L, M, \text{etc.}$ ) or the SI units (m, kg, *etc.*).
- (b) Is the equation correct numerically (within experimental error)? To answer this question, you will need to find numerical values for the various terms on the left hand side. You may use any resource you think is reliable, but you must cite your source. (Wikipedia is a good place to start.)
- (c) Which of the four “physical constants” in the equation do you think is known with the highest precision? Which do you think has the largest experimental uncertainty? Explain your answers.
- (d) What, if any, point do you think that Munroe was trying to make with this cartoon?
2. The nucleus of a typical human cell is roughly spherical with a diameter of about  $5\ \mu\text{m}$ . The nucleus contains, among other things, the DNA making up the genome, which for humans consists of 46 chromosomes containing a total of about  $3 \times 10^9$  basepairs (bp). The DNA molecules are roughly cylindrical (but flexible) with a diameter of approximately 2 nm and a length of 0.34 nm per base pair.
- (a) What is the volume of a typical nucleus? Express your answer in both  $\mu\text{m}^3$  and liters.
- (b) What is the total length of the DNA making up the human genome? What volume does the DNA occupy? What fraction of the cell nucleus is occupied by DNA?
- (c) An *E. coli* bacterium is roughly a cylinder  $1\ \mu\text{m}$  in diameter and  $2\ \mu\text{m}$  long. Its genome contains approximately  $5 \times 10^6$  bp of DNA. What is the volume occupied by the *E. coli* genome? What fraction of the cell volume does the DNA occupy?
3. Consider the game of craps, played with two six-sided dice. The analysis of this game is actually rather complicated, and we won't attempt a full treatment here. The player, or shooter, first throws the two dice. If the sum of the two numbers that appear is either 7 or 11, the shooter wins, and the game is over. If the sum is 2, 3 or 12, the shooter loses and the game is over. If any of the other possible sums (4, 5, 6, 8, 9 or 10) appears, this number is set as the “point”. The shooter continues to throw the dice until either the point is thrown or a 7 appears. If the point appears, the shooter wins. If 7 appears, the shooter loses. For the following, assume that each die is “fair”, *i.e.* the probabilities of the numbers 1 to 6 appearing on each die are equal.
- (a) Write out the sample set,  $S$ , for a given throw of the dice. Define each possible outcome as an ordered pair representing the numbers that appear on each die, *e.g.* (1,1).
- (b) Define the 11 possible sums as events,  $E_2, E_3, E_4 \dots E_{12}$ . That is, event  $E_2$  includes all of the outcomes for which the sum is 2; event  $E$  includes all of the outcomes for which the sum is 3, and so on. For each event, write out the set of outcomes that the event includes. Calculate the probability of each event

- (c) For the first throw, define 8 possible events: win (Ew1), lose (El1) and each of the six possible point sets (EP4, EP5, EP6, EP8, EP9 and EP10). For each of these events, identify the set of outcomes it includes and calculate the probability. Show that these probabilities add up to 1.
- (d) If the first throw does not result in a win or loss, what point (or points) will give the shooter the best chance of eventually winning? Which point (or points) will give the shooter the worst chance of winning?
- (e) The game can, in principle, go on indefinitely. Suppose that the first throw sets the point as 8. What is the probability that the shooter will throw another 10 times without winning or losing? What is the probability of throwing another 100 times without winning or losing? Be sure to explain your reasoning.