Biology 3550: Physical Principles in Biology
Fall Semester - 2017

Problem Set 4

Due: Monday, 6 November 2016, by 11:59 PM

Note: Be sure to show your work and use the proper units. You are encouraged to consult one another, or other resources, but the work that you hand in must be your own! Your work should be typed up and submitted as a single pdf file.

1. In class we discussed the diffusion of water and CO_2 across the stomata of plants. With the colonization of Mars a current topic of discussion, it might be interesting to consider how plants that have evolved on Earth might manage in the very different atmosphere found on Mars.

   The atmosphere of Mars is much thinner than that of Earth, with a pressure of about 600 Pa at the planet's surface versus approximately 10^5 Pa (100 kPa) on Earth. (1 Pa = 1 N/m^2). However, CO_2 represents about 96% of the Martian atmosphere, versus less than 1% on Earth. The temperature of the atmosphere of Mars at its surface is about 1°C. The diffusion coefficients of molecules in the gaseous state are sensitive to both temperature and pressure, but the lower pressure on Mars has a much bigger effect than the lower temperature.

   For the following, assume that the diffusion coefficients of both CO_2 and water in the Martian atmosphere are 2×10^{-3} m^2/s. Also assume that the diameter of the open stomata is 10 μm and that the length of the stomatal pores is 40 μm.

   (a) Using the information provided above (and the ideal gas law), calculate the molar concentration of CO_2 in the atmosphere at the surface of Mars.

   (b) Assuming, as we did in class, that the CO_2 concentration in the leaf airspace of a plant taken to Mars is about 1/2 of that in the atmosphere, calculate the CO_2 concentration gradient across a stoma (singular of stomata) and the flux, J, of CO_2 across the stomata.

   (c) Calculate the number of open stomata needed to transport 1 kg of carbon into a plant on Mars in one Earth year, with the stomata open half of the time. How does this compare to the number we calculated for a plant on Earth?

   (d) In order for a plant to remain alive in the Martian atmosphere, the cells and their immediate surroundings would have to remain liquid, and water vapor would be able to diffuse out of the plants through the stomata. If the plants were kept at 5°C, to avoid freezing, the maximum vapor pressure of water would be 0.87 kPa. Assuming that this is the water vapor pressure within the airspace of the plants, calculate the molar water concentration in the airspace.

   (e) Assuming that the water vapor concentration just outside of the plant leaves is one half of that in the airspace, calculate the flux, J, of water molecules through the stomata.

(f) Calculate the loss of water, over one year, through the open stomata that would be required to allow fixation of 1 kg of CO_2 on Mars.

(g) Your calculations should lead to a rather optimistic view of growing plants on Mars. But, what big factor has been ignored here? (What other consequence will there be of the very low atmospheric pressure?)

2. The admittedly abstract discussion in class of gas expansion and compression is not completely without practical significance: Consider the work required to pump up a bicycle tire:

(a) Calculate the volume of air within a typical bicycle tire (or, if you like, choose an atypical tire size that you prefer). This calculation does not need to be precise: A reasonable approximation would be the volume of a cylinder with the radius of the inner tube and the length of its circumference.

(b) Calculate the number of moles of air molecules in the inflated tire. For this calculation, it is reasonable to treat the air as an ideal gas and use the gas constant with units of atmospheres for pressure and liters for volume:

$$R = 0.08206\,\mathrm{L \cdot atm \cdot K^{-1} mol^{-1}}$$

Assume that the temperature is 300 K, and be sure to indicate the final pressure that you are assuming. What is the initial volume of the air at atmospheric pressure?

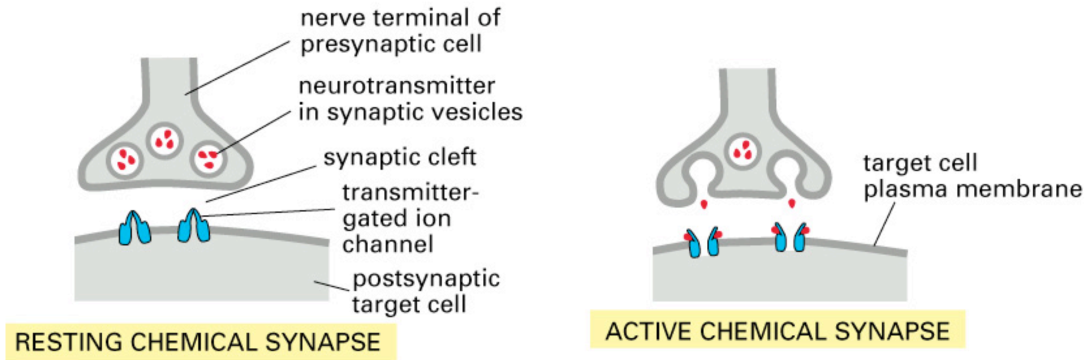
(c) Calculate the entropy change, at 300 K, for compressing the air from 1 atm to the final pressure.

(d) How would you go about filling the tire with the minimum amount of work? What conditions would you maintain during this process? What is the minimum amount of work required?

(e) If you have much experience with this activity, you probably know that the temperature of the air is likely to increase as you fill the tire. Why does the temperature increase? How would you expect the temperature increase to affect the amount of work that you have to do?

(f) Suppose that you have just eaten a candy bar. How many tires must you pump up to work off the nutritional energy of the candy bar, assuming that 10% of the candy bar calories are converted to work (and the rest to metabolic heat)? Also assume that the temperature of the air remains constant and that you use the minimum amount of work to pump up each tire. Be careful: nutritional information is usually reported in "big C" calories, which is the amount of energy required to raise the temperature of 1 Kg of water by 1 °C, not 1 g.

3. Synapses are special structures through which a neuron (the presynaptic cell) communicates with either another neuron or a muscle cell (the postsynaptic cell), as illustrated in the figure below (from Alberts *et al.*, Molecular Biology of the Cell):



The presynaptic cell contains vesicles filled with small molecules (neurotransmitters, such as acetylcholine, glutamate or serotonin). When triggered, these vesicles fuse with the cell membrane and release their contents into the space separating the presynaptic and postsynaptic cells. After diffusing to the postsynaptic cell, the neurotransmitter molecules bind to receptors and trigger a signal.

Suppose that one of the vesicles of a presynaptic cell has a volume of 10^{-19} L and contains 10^4 neurotransmitter molecules. Also assume that the volume of the space between the two cells, the synaptic cleft, is 10^{-14} L.

(a) Calculate the entropy change associated with the release of 10^4 neurotransmitter molecules from a vesicle into the synaptic cleft.

(b) What is the minimum amount of work (in Joules) that would be required to collect up the neurotransmitter molecules and put them back into a vesicle at their original concentration, at 37 °C?

(c) The standard free energy change for the hydrolysis of ATP is approximately -30 kJ/mol. Assuming that this value represents the amount of work that can be obtained by the hydrolysis of 1 mole of ATP under physiological conditions[1], how many molecules of ATP would be required to repackage the contents of one vesicle? Assume that the energy of ATP hydrolysis is used with perfect efficiency.

[1] Things are more complicated than this, as we will discuss shortly