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Biology 3820
Physical Principles in Biology
Spring Semester 2011

Mid-Term Exam
18 March 2011

Please write your name on each page.
Be sure to show your work and include correct units in all of your answers!

Some possibly useful constants:
The gas constant: $8.314\,\mathrm{J}\cdot\mathrm{mol}^{-1}\mathrm{K}^{-1}$
Avogadro's number: 6.02×10^{23}

1. (30 pts) Two students are exploring probability by tossing coins. Each experiment consists of tossing a coin ten times, and the result is defined in terms of the sequence of heads and tails for the individual tosses. They repeat this experiment 100 times. (Afterwards, they decide that maybe it would be worthwhile to learn Python and use a computer to simulate the process.)

   (a) How many possible outcomes (sequences of heads and tails) are there for a single experiment (10 coin tosses)?

   (b) Assuming that the coin is fair, which of the possible outcomes is most likely?

(c) Suppose that the coin was, in fact, biased, so that the probability of heads for any individual toss is 0.37. Which of the possible outcomes is the most likely?

(d) Assuming, once again, that the coin is fair, what is the probability of observing six heads in a single experiment?

(e) After doing 100 experiments, and recording the sequences, the students note that five of the sequences include runs of 6 heads in a row. One student thinks that this is very suspicious, but the other says that such runs are to be expected. What do you think? Explain your answer as quantitatively as you can.

2. (25 pts) An entomologist, Dr. Shirley Bugsmee, is studying the behavior of ants of a particular species. She places an ant at the middle of a grid that is completely isolated from any food or other substances the ants can detect, and then watches its behavior. It appears to her that the ant is taking a random walk in which it moves forward some distance, changes direction randomly and then repeats itself. She has recruited several students to help in her experiments, and together they follow the behavior of 100 ants, each for a total of 50 min. They find that the average time between changes in direction is 5 s, and the average distance the ants move away from their starting position in 50 min is 0.25 m.

(a) Assuming that the ants really are taking a random walk, what is the average distance they move between changes in direction?

(b) What is the total distance (*i.e.* the sum of all of the steps) that the ants move in 50 min?

(c) At what location is an ant most likely to end up after 50 min? If you answer this question without doing any calculations, explain your reasoning.

(d) Suppose the ants were allowed to walk for 150 min. What would you expect their average distance from the starting position to be?

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3. (30 pts) A bacterium is working valiantly to survive on glycerol as its sole carbon source. The glycerol concentration in the surrounding medium is 1 mM, while the bacterium must maintain an intracellular concentration of 50 mM in order to feed its metabolism. An ATP-driven "pump" actively transports glycerol into the cell. In addition to the glycerol consumed by metabolism, glycerol can leave the cell by passive diffusion across the lipid bilayer of the cell membrane. The permeation coefficient for the diffusion of glycerol across the bilayer is $10^{-8}\,\mathrm{ms}^{-1}$. For the following, assume that the bacterium is a cylinder 2 μm long and 1 μm in diameter, and that the thickness of the bilayer is 2 nm. The bacteria are growing at 37 °C.

   (a) What is the flux, in appropriate dimensions, of glycerol out of the bacterium due to diffusion across the bilayer?

   (b) What is the net number of molecules of glycerol that diffuse out of the cell per minute?

(c) Assuming that the intracellular glycerol concentration remains constant at 50 mM, what is the free energy change associated with the diffusion of a single molecule of glycerol out of the cell?

4. (15 pts) The standard free energy change for the hydrolysis of ATP is approximately -30 kJ/mol. Suppose that, in the bacterium described in the previous problem, the concentration of ATP and ADP are 10 mM and 1 mM, respectively, and the concentration of inorganic phosphate is 2 mM.

(a) What is the free energy change for ATP hydrolysis under these conditions ?

(b) Under the conditions specified above, how many molecules of ATP per minute would be required to pump back into the cell the glycerol that diffuses out across the bilayer? Assume that the energy of ATP hydrolysis is used with perfect efficiency.